Name: ______________________________

Geometry CC Survival Guide

Common Core High School Math Reference Sheet
(Algebra I, Geometry, Algebra II)

CONVERSIONS

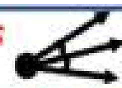
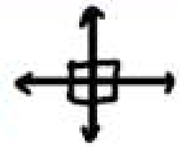
1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

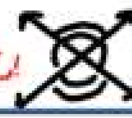
FORMULAS

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1-r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t-t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		

Unit 1: Fundamentals of Geometry

Vocabulary

- *Bisect:* Cut into two congruent parts 
- *Perpendicular Lines:* two lines that intersect and form right angles 
- *Linear Pair:* two angles that form a line 

- *Complementary:* angles that add to 90°
- *Supplementary:* angles that add to 180°
- *Vertical Angles:* angles formed by two intersecting lines; ALWAYS EQUAL! 

SAME

Formulas- MUST KNOW THESE!

Slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

*Used to determine if lines are PARALLEL, PERPENDICULAR, OR NEITHER!

Parallel Lines: SAME slopes

Perpendicular Lines: NEGATIVE RECIPROCAL slopes (flip & negate)

*Used to determine if lines create right angles.

Show that the slopes of the lines are…

NEGATIVE RECIPROCALS →perpendicular lines → right angles.

Midpoint:

$$mdpt = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

*Used to determine if segments were BISECTED.

If two segments intersect at the same midpoint, then the segments bisect each other.

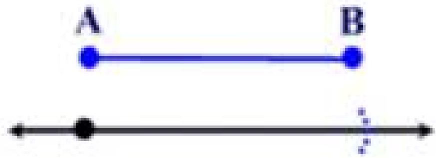
Distance:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

*Used to determine the LENGTH of a segment.

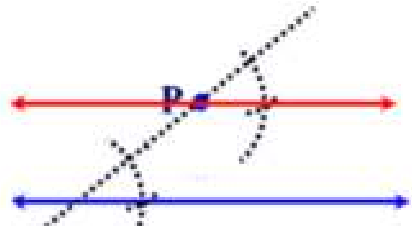
Constructions

Copy a Segment



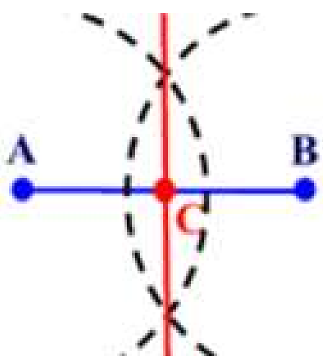
mathopenref.com/constcopysegment.html

Parallel Lines



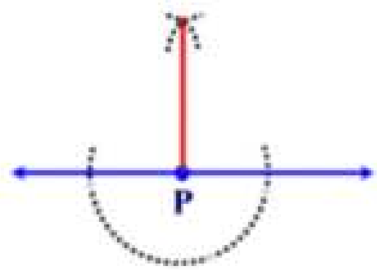
mathopenref.com/constparallel.html

Perpendicular Bisector



mathopenref.com/constbisectline.html

Perpendicular Line Through Given Point ON the Line



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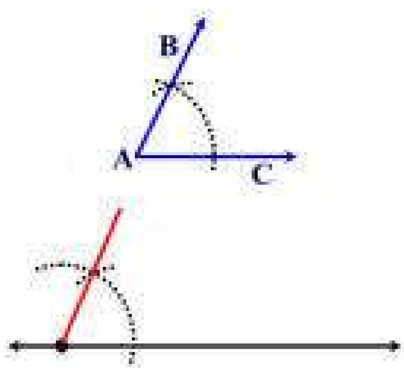
Copy an Angle



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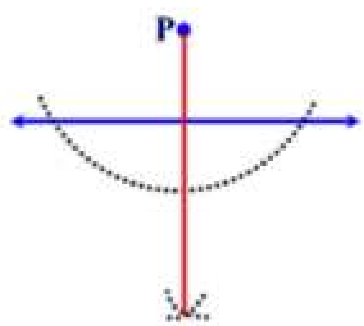
Perpendicular Line Through Given Point NOT ON the Line



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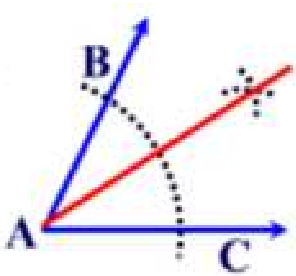
Angle Bisector



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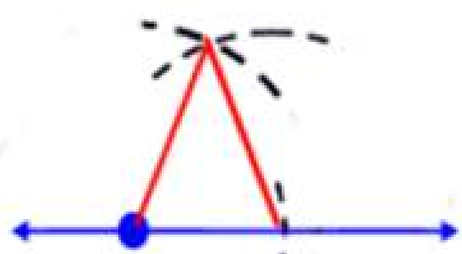
Isosceles Triangle



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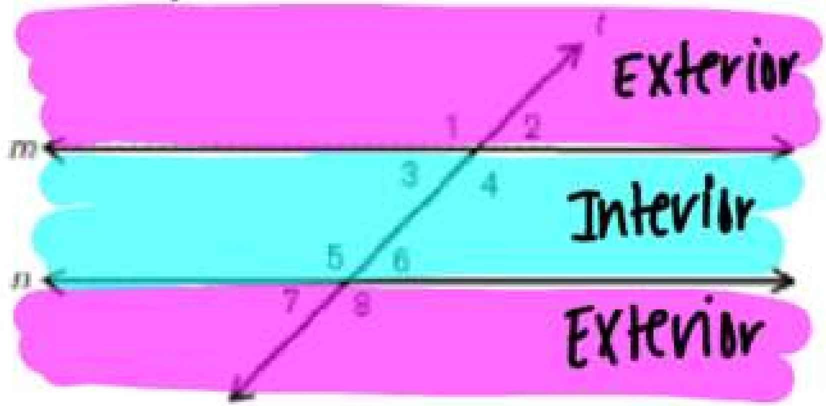
Parallel and Perpendicular Lines

I. *Parallel Lines Cut By a Transversal*



If $m||n$, then…

- Alternate interior angles are congruent.

  $\angle 3 \cong \angle 6$

  $\angle 4 \cong \angle 5$

- Alternate exterior angles are congruent.

  $\angle 1 \cong \angle 8$

  $\angle 2 \cong \angle 7$

- Corresponding angles are congruent.

  $\angle 1 \cong \angle 5$ $\angle 3 \cong \angle 7$

  $\angle 2 \cong \angle 6$ $\angle 4 \cong \angle 8$

- Same Side Interior angles are supplementary.

  $m\angle 3 + m\angle 5 = 180^\circ$

  $m\angle 4 + m\angle 6 = 180^\circ$

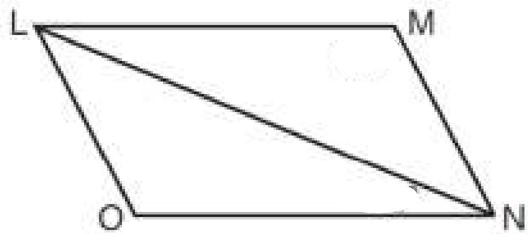
**Note: The converses of the above statements are also true!*

(change order)

Examples:

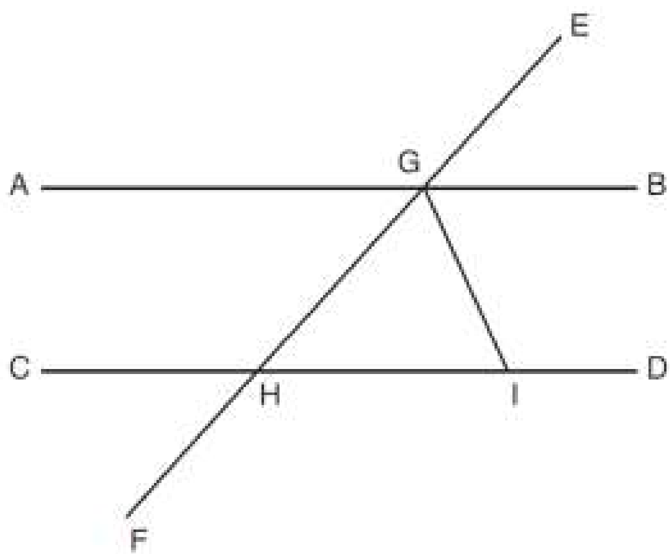
1. The diagram below shows parallelogram LMNO with diagonal $\overline{LN}, m < M = 120°$, and $m < LNO = 20°$.

Need help?
Scan:



Explain why m$\angle NLO$ is 40 degrees.

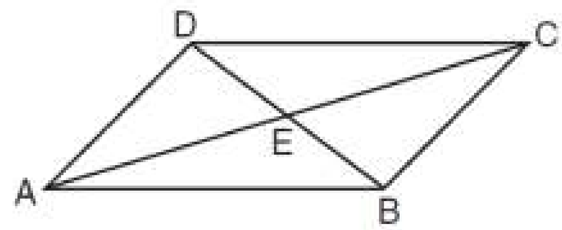
2. In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m < EGB = 40°$ and $m < DIG = 110°$, explain why $\overline{AB} || \overline{CD}$.

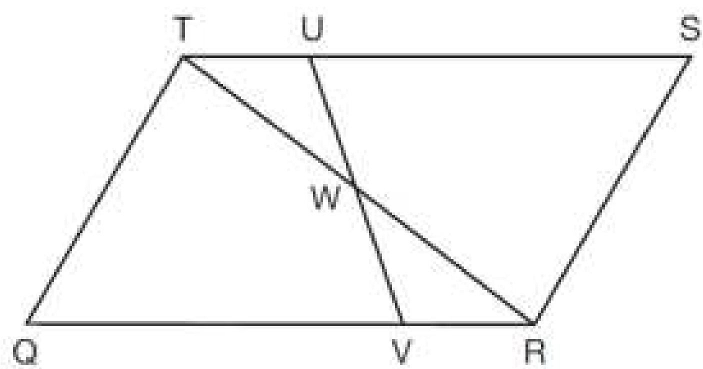
Need help? Scan:

3. In parallelogram $ABCD$ shown below, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E.



Prove: $< CDB \cong < ABD$

4. In parallelogram $QRST$ shown below, diagonal $\overline{TR}$ is drawn, U and V are points on $\overline{TS}$ and $\overline{QR}$, respectively, and $\overline{UV}$ intersects $\overline{TR}$ at W.



If $m < S = 65°, m < SRT = 73°,$ and $m < TWU = 38°$, what is $m < WVQ$?

II. Parallel Lines in the Coordinate Plane

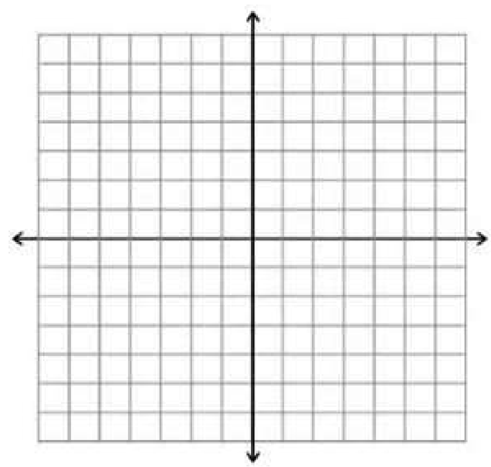
- SLOPES OF PARALLEL LINES

Need help? Scan:

Parallel lines have EQUAL slopes.

- *Example:* What is the slope of a line that is parallel to the line whose equation is $2x+3y=6$?

- How do we write the equation of a line parallel to a given line that passes through a specific point?
  - *Example:* Write the equation of a line that is parallel to the line whose equation is $4x + 3y = 7$ and also passes through the point (-6, 2)?



Need help? Scan:

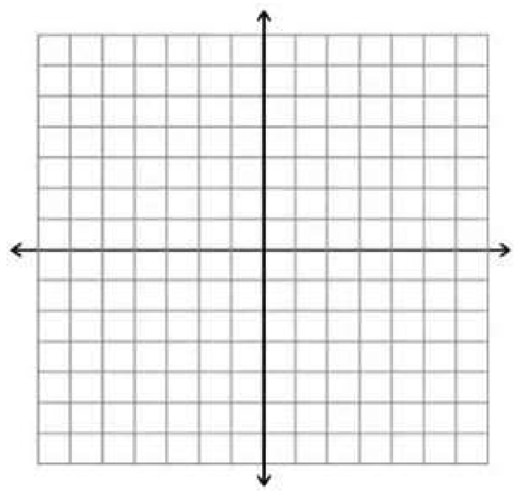
III. Perpendicular Lines in the Coordinate Plane

- SLOPES OF PERPENDICULAR LINES

Perpendicular lines have **NEGATIVE RECIPROCAL** slopes.

- *Example:* What is the slope of a line that is perpendicular to the line whose equation is $3x + 5y = 4$?

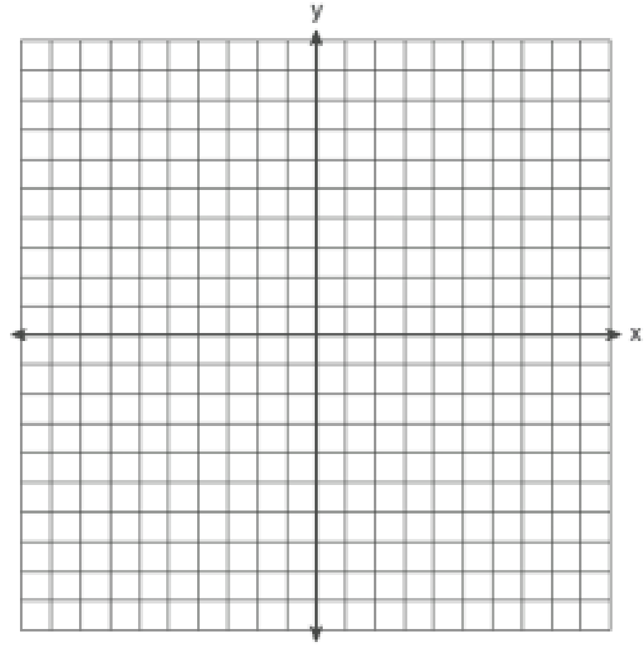
- How do we write the equation of a line perpendicular to a given line that passes through a specific point?
  - *Example:* What is an equation of the line that contains the point $(3, -1)$ and is perpendicular to the line whose equation is $y = -3x + 2$?



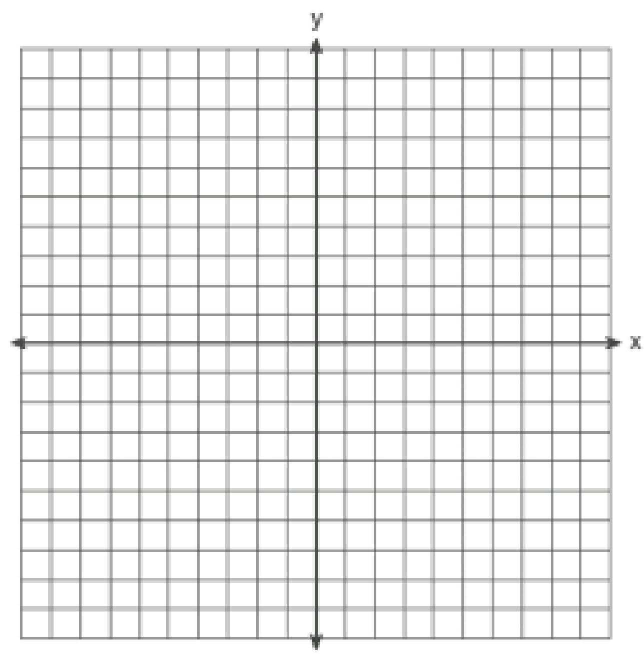
Need help?
Scan:

Partitioning a Segment

Ex 1: Point P partitions directed line segment $\overline{AB}$ with A(-2, -5) and B (6, 1) into a ratio of 1:3. Find the coordinates of point P.



Ex 2: Line segment $\overline{AB}$ has endpoints A (3, 4) and B (6, 10). Find the coordinates of point P along the directed line segment line segment $\overline{AB}$ so that $AP:PB = 3:2$



Unit 2: Transformations

- **Symmetry**
  - Point: Turn upside down ($R_{180°}$) & see if figure looks the same
  - Line: Fold figure & see if the pieces match up
  - Rotational: Turn figure any number of degrees (less than 360) & see if the figure looks the same
- **Isometry:** distance/lengths stays the same
  - Opposite: arrows opposite direction
  - Direct: arrows same direction

RIGID MOTIONS: Translations, Reflections, & Rotations

TRANSFORMATIONS

- **Translations:** Slide a given distance and direction.

$T_{a,b}$

ADD "a" to x-values & "b" to y-values or MOVE right/left a units and up/down b units

- **Rotations:**

R_{angle}

1. Plot **ORIGINAL** coordinates
2. Turn paper:
   Positive Angles- *turn counterclockwise*
   Negative Angles- *turn clockwise*
3. Read & record **NEW** coordinates
4. Plot **NEW** coordinates

To construct the center of rotation: Construct two perpendicular bisectors of any two points and their images. The point where the two perpendicular bisectors meet is the center of rotation

To find the degree of rotation: Measure the angle formed by any point, the center of rotation (vertex) and the image of that point.

- **Reflections:** COUNT # of units TO the line & go the # of units FROM the line

r_{line}

The **Line of Reflection** is the perpendicular bisector of any point and its image.

To construct the line of reflection, join any point and its image. Then construct the perpendicular bisector of that segment.

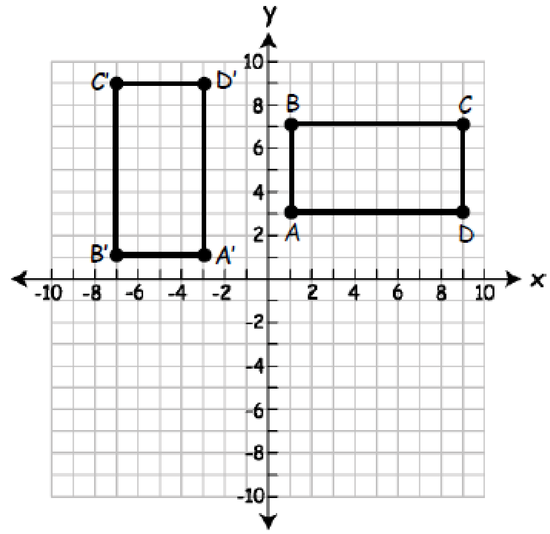
- **Compositions of Transformations**

****DO BACKWARDS! (Right first, then left)****

Need help?
Scan:

Practice Problems:

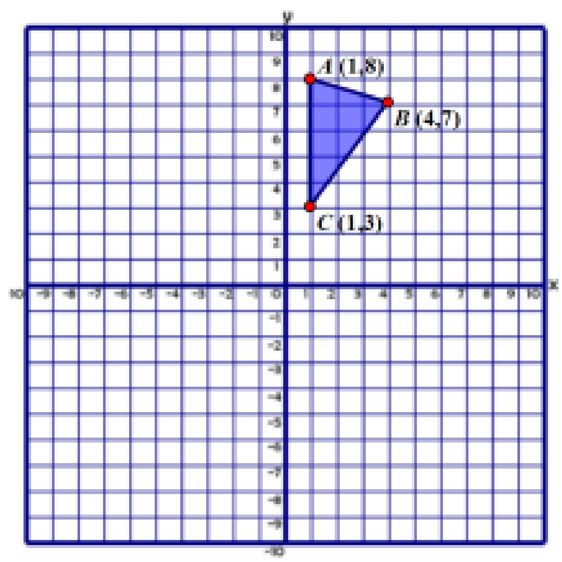
1. Based upon the figure below, describe how rectangle *ABCD* can be carried onto its images *A'B'C'D'*.



(1) Reflection across the x-axis
(2) Reflection across the y-axis
(3) Rotation 90° clockwise about the origin
(4) Rotation 90° counterclockwise about the origin

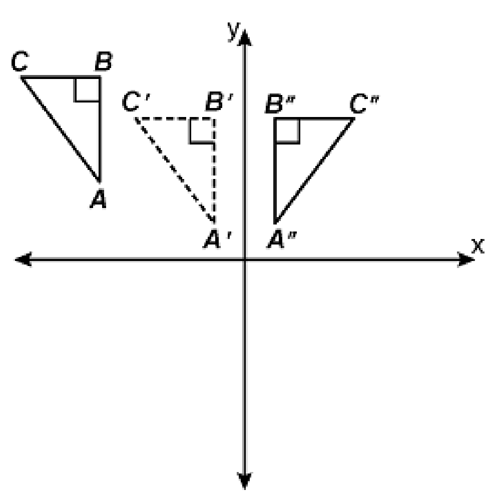
2. Which single transformation is equivalent to $r_{y-axis} \circ r_{x-axis}$?

(1) $R_{90°}$
(2) $r_{y=x}$
(3) $T_{(-2,-16)}$
(4) $R_{180°}$



3. In the diagram below, ΔA'B'C' is a transformation of ΔABC, and ΔA''B''C''; is a transformation of ΔA'B'C'. The composite transformation of ΔABC to ΔA''B''C'' is an example of a

(1) first a translation, then a reflection
(2) first a reflection, then a rotation
(3) first a reflection, then a translation
(4) first a translation, then a rotation

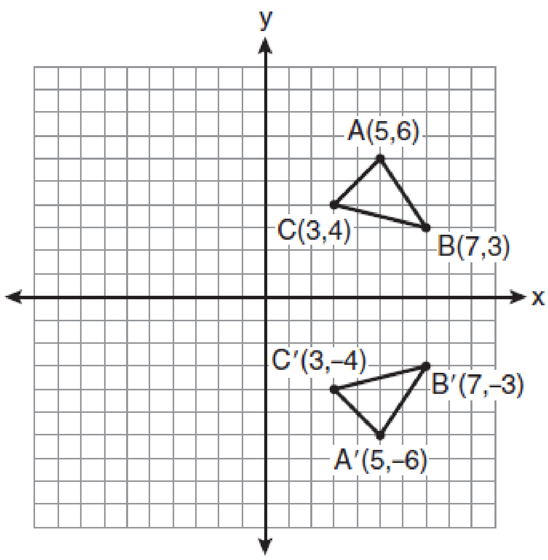


Need help? Scan:

4. The vertices of parallelogram *ABCD* are $A(2,0)$, $B(0,-3)$, $C(3,-3)$, and $D(5,0)$. If *ABCD* is reflected over the x-axis, how many vertices remain invariant?

1) 1
2) 2
3) 3
4) 0

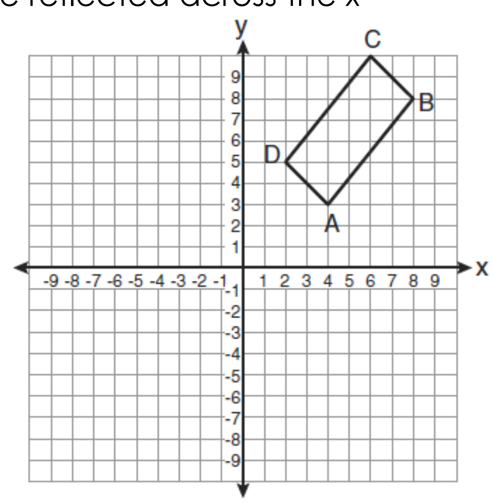
5. Which expression best describes the transformation shown in the diagram below?



1) same orientation; reflection
2) opposite orientation; reflection
3) same orientation; translation
4) opposite orientation; translation

6. The rectangle *ABCD* shown in the diagram below will be reflected across the x-axis. What will *not* be preserved?

1) slope of $\overline{AB}$
2) parallelism of $\overline{AB}$ and $\overline{CD}$
3) length of $\overline{AB}$
4) measure of $\angle A$



7. Triangle ABC has coordinates A (-3, 1), B (0, 5) and C (-5, 7).

Need help? Scan:

a) Sketch and state the coordinates of ΔA'B'C', **the image of ΔABC** after $r_{x=2}$

A' (,) B' (,) C' (,)

b) Graph and state the coordinates of ΔA''B''C'', **the image of ΔA'B'C'** after $\langle -10, -7 \rangle$.

A'' (,) B'' (,) C'' (,)

c) Graph and state the coordinates of ΔA'''B'''C''', **the image of ΔA''B''C''** after $R_{90°}$.

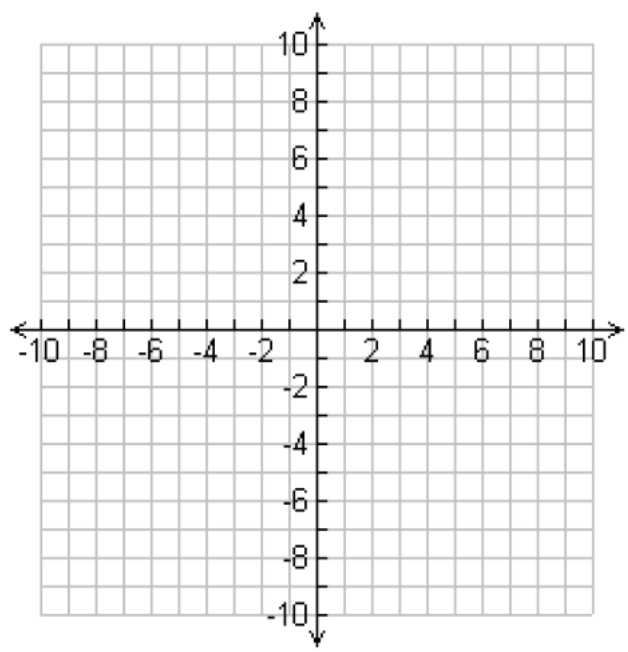
A''' (,) B''' (,) C''' (,)

d) Which transformation does ***not*** preserve orientation?

(1) $r_{x=2}$

(2) $\langle 2, -7 \rangle$

(3) $R_{90°}$



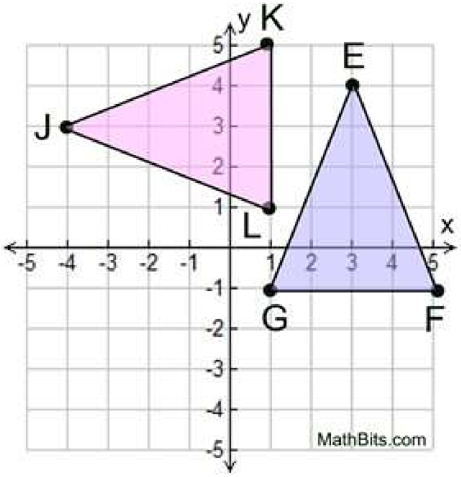
Proving Congruent Triangles

Key Idea #1: Two figures are congruent if and only if there exists a sequence of rigid motions that will map one figure onto the other

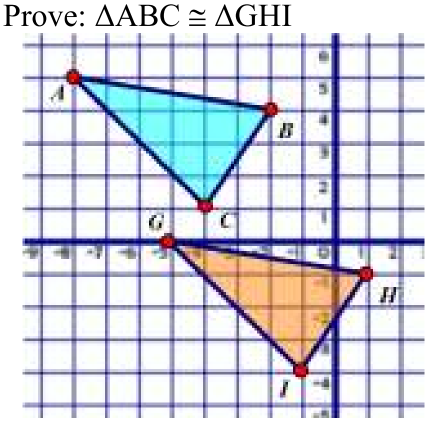
Examples:

Need help? Scan:

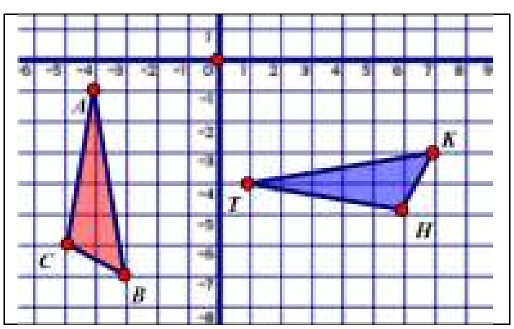
1. Which **specific** rigid motion could be used to prove $\Delta EFG \cong \Delta JKL$?



2. Prove: $\Delta ABC \cong \Delta GHI$



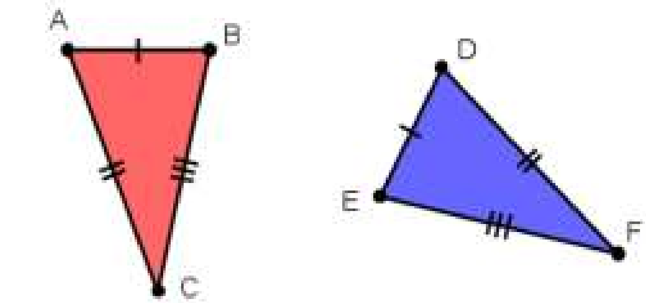
3. Prove: $\Delta ABC \cong \Delta TKH$



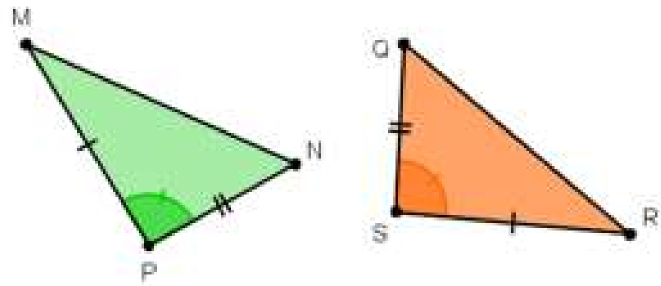
Key Idea #2: There are 5 Triangle Congruence Theorems that may be used to prove two congruent triangles.

TRIANGLE CONGRUENCE THEOREMS:

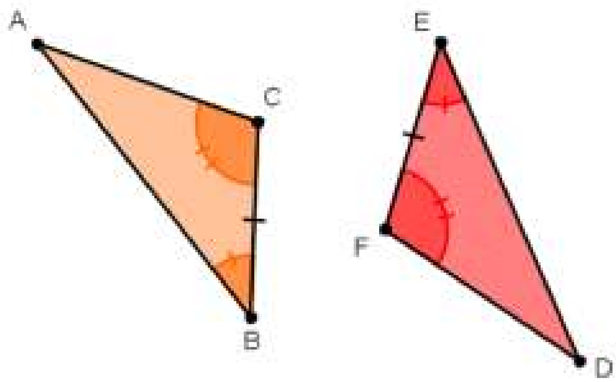
SSS



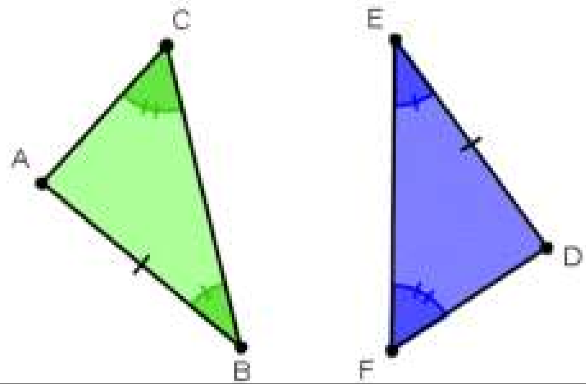
SAS



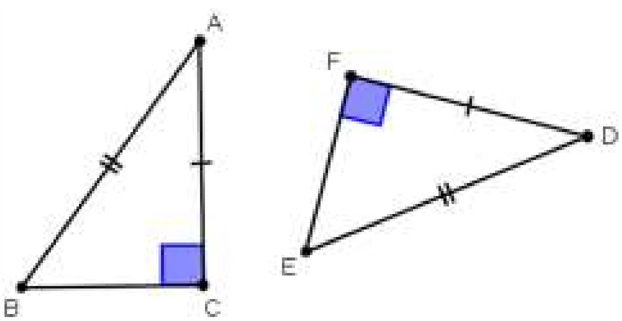
ASA



AAS or SAA



HL



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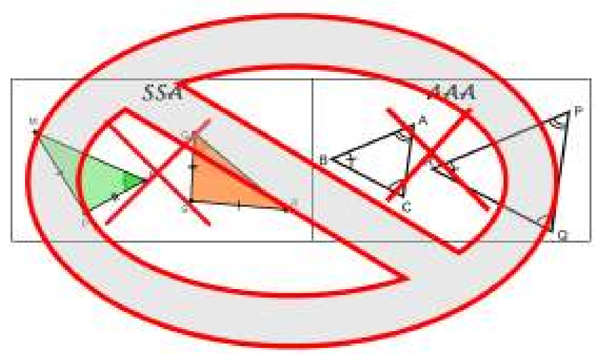


TRIANGLE CONGRUENCE PROOFS

Always remember to <u>MARK</u> <u>YOUR</u> <u>DIAGRAM</u>! ☺

<u>When Triangles Overlap….</u>
SEPARARATE the triangles and look for shared sides and/or angles

<u>Use the diagram to find…..</u>
1.) Vertical angles
2.) Shared sides (Reflexive property)
3.) Supplementary angles (Linear Pairs make supp angles)
4.) Shared angles (Reflexive property)
5.) Isosceles Triangles (Look for 2 ≅ sides or 2 ≅ angles in the same triangle)
6.) In circles… look for congruent radii, congruent diameters, and inscribed angles cutting into the same arc
7.) In parallelograms… look for parallel lines and "Z" shapes because // lines make ≅ alternate interior angles

<u>"Reasons" to use in statements involving line segments…..</u>
Midpoint makes 2 congruent segments
Bisector makes 2 congruent segments (for segment bisector)
Segment Addition Postulate (Equals added to Equals are Equal)
Segment Subtraction Postulate (Equals subtracted from Equals are Equal)
Altitude starts at vertex and is perpendicular to the opposite side (or extension of opposite side)
Median connects vertex to midpoint
Perpendicular Bisector passes through midpoint and is perpendicular to given segment

<u>"Reasons" to use in statements involving angles……..</u>
Perpendicular lines form right angles
All right angles are congruent
Vertical angles are congruent
Bisector makes 2 congruent angles (for angle bisector)
Angle Addition Postulate(Equals added to Equals are Equal)
Angle Subtraction Postulate (Equals subtracted from Equals are Equal)
When two angles in one triangle are ≅ to two angles is another triangle, the 3rd angles are also ≅

<u>"Reasons" to use in statements involving parallel lines…….</u>

// lines make ≅ alternate interior angles
// lines make ≅ alternate exterior angles
// lines make ≅ corresponding angles
// lines make SSI (same side interior) angles supp

≅ alternate interior angles make // lines
≅ alternate exterior angles make // lines
≅ corresponding angles make // lines
Supp SSI (same side interior) angles make // lines

2 lines // to the same line are // to each other
2 lines ⊥ to the same line are // to each other

<u>When using Congruent Supplements Theorem, (</u>≅ angles have ≅ supp)
<u>you must discuss:</u>

1.) ≅ Angles
2.) Supplementary Angles

<u>"Reasons" to use in proofs involving isosceles triangles…..</u>

When a triangle has 2 ≅ sides, the angles opposite those sides are also ≅
When a triangle has 2 ≅ angles, the sides opposite those angles are also ≅

<u>"Reasons" to use in Proving ≅ Triangles ……</u>

SSS
SAS
ASA
AAS
HL-Rt. Δ (Remember that you must write about right **TRIANGLES** when using this method)

<u>CPCTC…..</u>

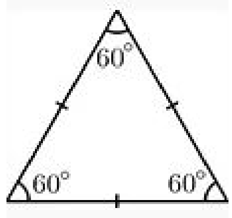
*USE CPCTC WHENEVER YOU A TRYING TO PROVE A <u>PAIR OF CONGRUENT ANGLES</u> OR <u>CONGRUENT SEGMENTS</u>

1.) Must prove ≅ triangles FIRST
2.) Then use CPCTC to get ≅ sides or ≅ angles

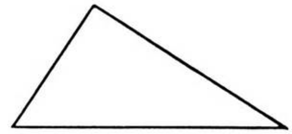
Unit 4: Triangles

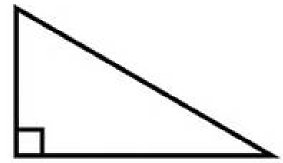
I. Types of Triangles

Equilateral
3 equal sides,
3 equal angles

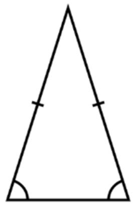


Scalene
no equal sides,
no equal angles





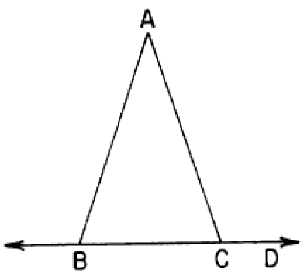
Isosceles
2 equal sides (legs),
2 equal base angles



Right
exactly one right angle,
either 2 equal sides (isosceles)
or no equal sides (scalene)

Examples:

1. In the accompanying diagram, $\overleftrightarrow{BCD}$, $\overline{AB} \cong \overline{AC}$, and m<A = 30. What is $m\angle ACD$?



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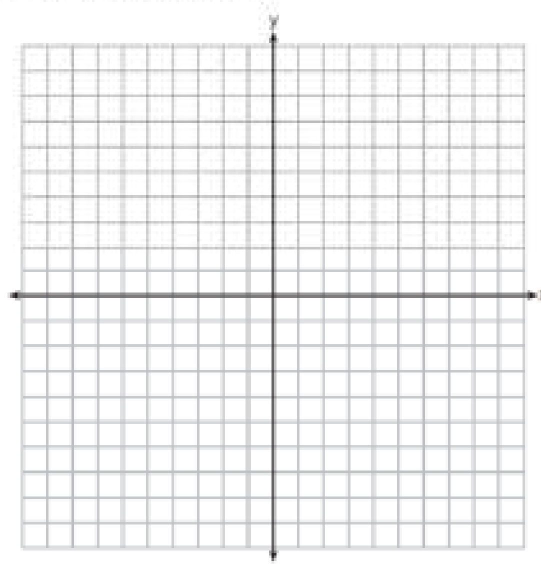
2. The coordinates of the vertices of $\triangle RST$ are $R(-2,-3)$, $S(8,2)$, and $T(4,5)$. Which type of triangle is $\triangle RST$?

(1) right (3) obtuse
(2) acute (4) equiangular



Coordinate Proofs:

1.) Show a triangle is equilateral by using distance formula to demonstrate that 3 sides are ≅
2.) Show a triangle is isosceles by using distance formula to demonstrate that 2 sides are ≅
3.) Show a triangle is scalene by using distance formula to demonstrate that no sides are ≅
4.) Show a triangle is a right triangle by using the distance formula and demonstrating that the 3 side lengths satisfy the Pythagorean Theorem (remember to use <u>longest</u> <u>side</u> for hypotenuse)

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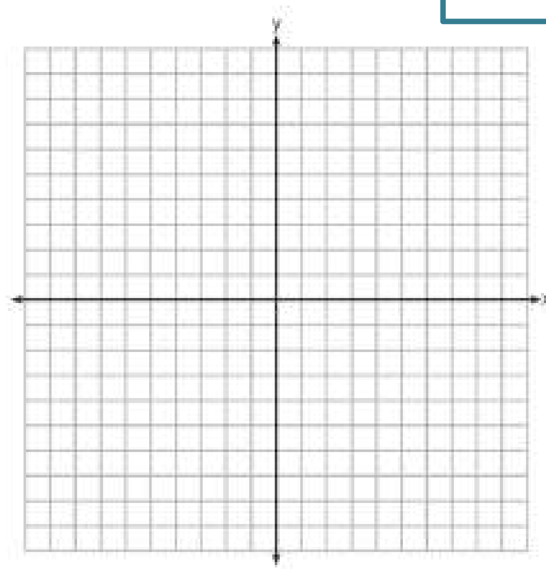
Example:

Triangle ABC has vertices with A(5, 6), B(x,5), and C(2, -3).
Determine and state a value of x that would make triangle ABC a right triangle. Justify why $\triangle ABC$ is a right triangle.

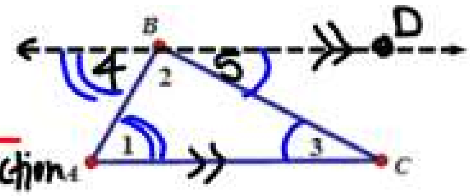


II. Interior Angles of Triangles

THE SUM OF ALL THE ANGLES IN A TRIANGLE IS $\underline{180^\circ}$!

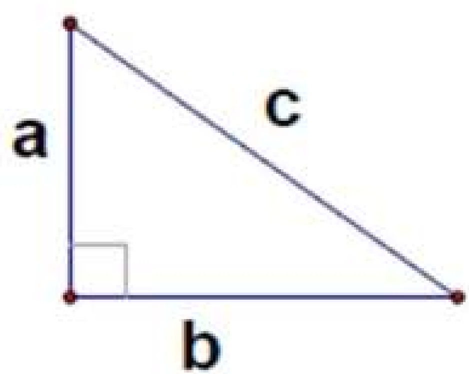
Proof:

Given: ΔABC
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statements	Reasons
① ΔABC, construct Auxiliary line $\overline{BD}$ so $\overline{BD} \parallel \overline{AC}$	① Given/construction
② $\angle 4, \angle 2, \angle 5$ are supplementary	② $\angle$'s that form a straight line are supp.
③ $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	③ supp. $\angle$'s add to 180°
④ $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	④ $\parallel$ lines $\rightarrow \cong$ alt. int. $\angle$'s
⑤ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	⑤ substitution

III. Pythagorean Theorem



$$a^2 + b^2 = c^2$$

USES:

- May be used to find the missing side of a RIGHT TRIANGLE
- May be used to determine if a triangle is a RIGHT TRIANGLE

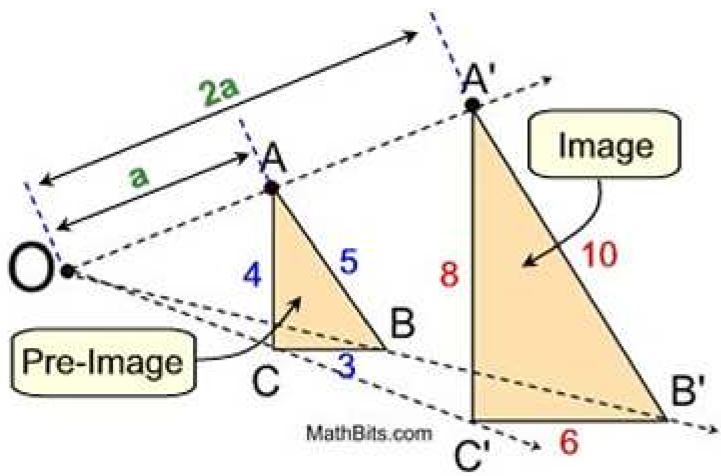
Unit 5: Similarity

DILATIONS

Dilation: a transformation that produces an image that is the **same shape** as the original, but is a **different size.**

DILATIONS ARE <u>NOT</u> RIGID MOTIONS!

Dilations of 2D Shapes



In this dilation, of scale factor 2 mapping ΔABC to $\Delta A'B'C'$, the distances from O to the vertices of $\Delta A'B'C'$ are twice the distances from O to ΔABC.

After a dilation, the pre-image and image have the same shape but not the same size.

Sides: In a dilation, the sides of the pre-image and the corresponding sides of the image are proportional.

$$\frac{image}{pre-image}: \frac{A'C'}{AC} = \frac{C'B'}{CB} = \frac{A'B'}{AB} = \frac{2}{1}$$

Dilations create similar figures!

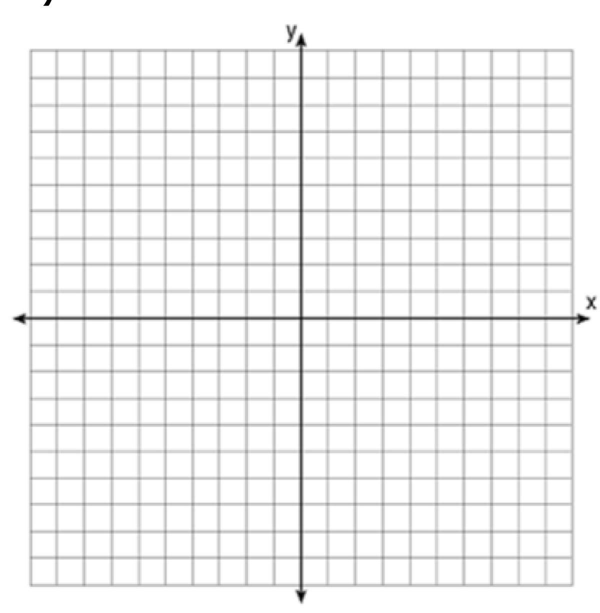
Properties preserved under a **dilation** from the pre-image to the image.
1. angle measures (remain the same)
2. parallelism (parallel lines remain parallel)
3. collinearity (points remain on the same lines)
4. orientation (lettering order remains the same)
----
5. distance is **NOT** preserved (lengths of segments are NOT the same in all cases except a scale factor of 1).
A dilation is NOT a rigid transformation (isometry).

Scale Factor, *k*:

- If $k > 1$, enlargement.
- If $0 < k < 1$, reduction.
- If $k = 1$, congruence.

If $k < 0$, the image will be placed on the opposite side of the center and rotated 180º.

EXAMPLE: Triangle DEF has coordinates **D(5,5)**, **E(2,-1)**, and **F(6, -3)**. State the coordinates of the image of ΔDEF under $D_{(8,3),2}$.



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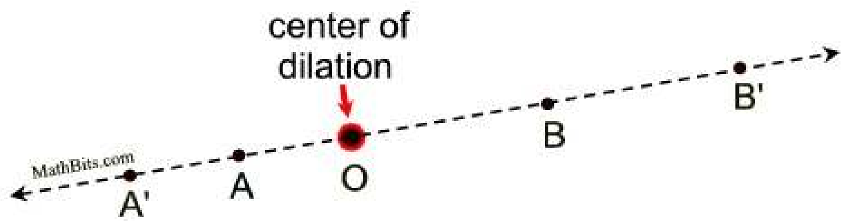
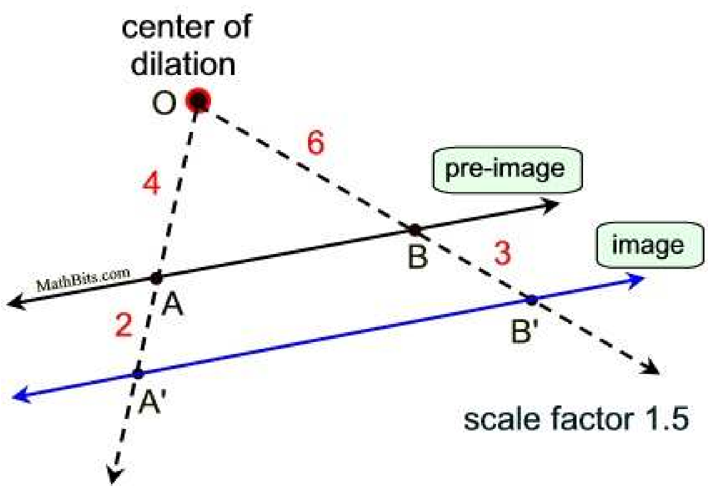
Dilations of Lines

Concept 1: A dilation leaves a line passing through the center of the dilation unchanged.

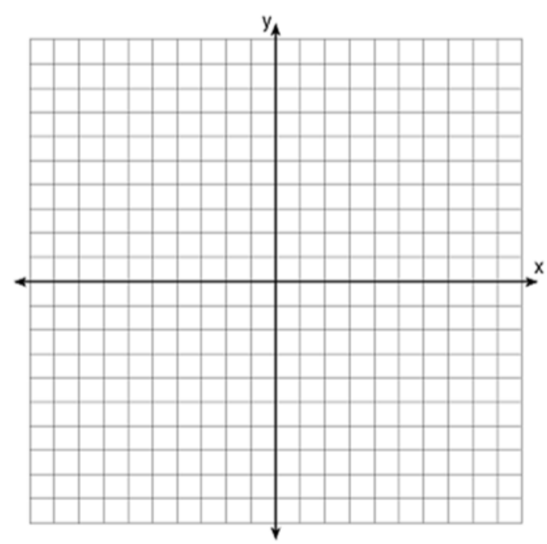


Concept 2: A dilation takes a line NOT passing through the center of the dilation to a parallel line.



EXAMPLES:

1. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. What is the equation of the line after the dilation?



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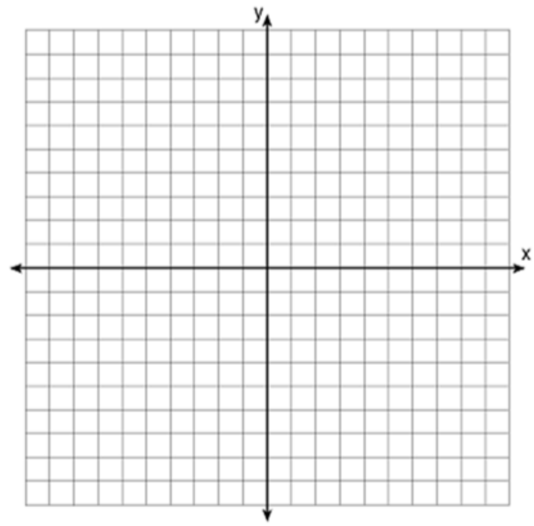
2. The line $y = 2x$ is dilated by a scale factor of 4 and centered at the origin. What is the equation of the line after the dilation?



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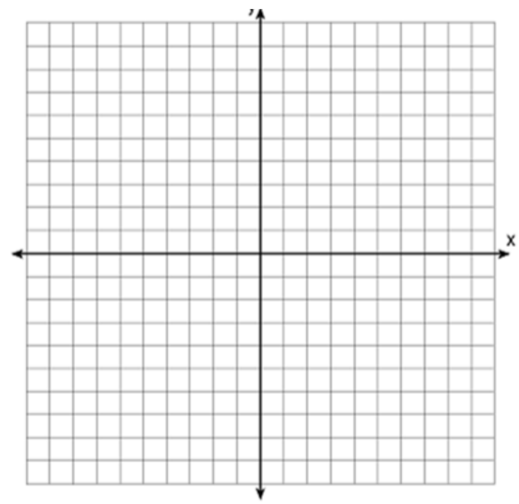
3. The line $y = 2x + 1$ is dilated by a scale factor of 2 and centered at $(-1,3)$. What is the equation of the line after the dilation?



4. Given line m and point O not on line m. The image of line m is constructed through a dilation centered at O with a scale factor of 3. Which of the following statements best describes the image of line m?
(1) a line passing through point O
(2) a line intersecting with line m
(3) a line parallel to line m
(4) a line perpendicular to line m

5. Line $\overleftrightarrow{AB}$ is dilated with a center of dilation at A and a scale factor of 2. Which of the following statements will be true about $\overleftrightarrow{AB}$ and its image $\overleftrightarrow{A'B'}$?
(1) The slope of $\overleftrightarrow{A'B'}$ will be twice the slope of $\overleftrightarrow{AB}$.
(2) The slope of $\overleftrightarrow{A'B'}$ will be half the slope of $\overleftrightarrow{AB}$.
(3) The slope of $\overleftrightarrow{A'B'}$ will be two more than the slope of $\overleftrightarrow{AB}$.
(4) The slope of $\overleftrightarrow{A'B'}$ will be the same as the slope of $\overleftrightarrow{AB}$.

6. A line is to be dilated. The center of dilation does not lie on the line. The scale factor is ½. Which of the following statements is TRUE about the resulting image of the line?
(1) The image is half the length of the line.
(2) The image is parallel to the line.
(3) The image is perpendicular to the line.
(4) The image intersects the line.

Need help?
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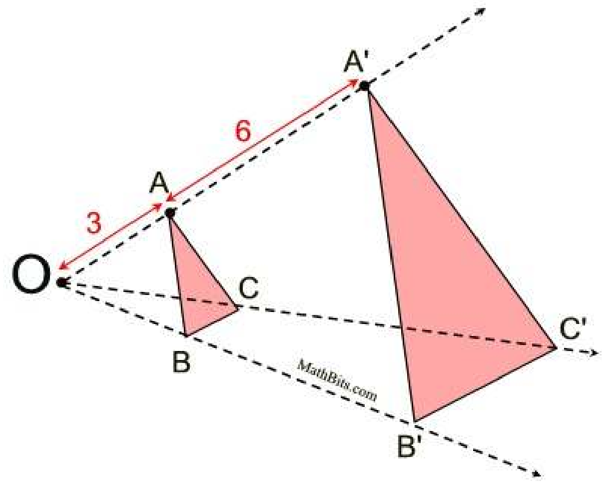
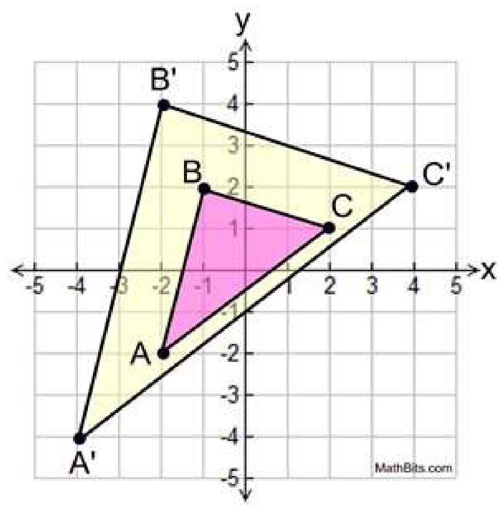
Finding the Scale Factor & Center of Dilation

Examples:

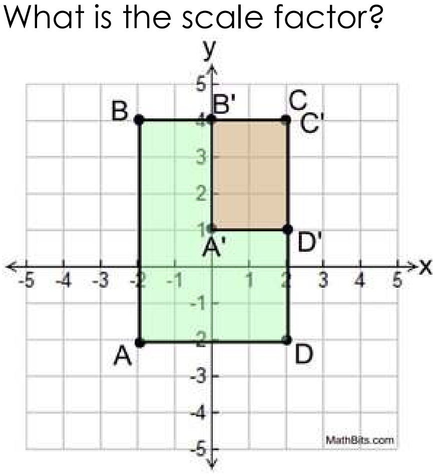
1. A dilation centered at *O* is shown at the right. The image is $\Delta A'B'C'$.
If $OA = 3$ units and $AA' = 6$ units, what is the scale factor of this dilation?



2. ΔABC has $A(-2,-2)$, $B(-1,2)$ and $C(2,1)$. After a dilation centered at the origin, $\Delta A'B'C'$ has $A'(-4,-4)$, $B'(-2,4)$ and $C'(4,2)$. What is the scale factor of the dilation?



3. Rectangle ABCD is dilated to create image A'B'C'D'. What is the center of dilation? What is the scale factor?



Similar Figures

Similar Figures: a correspondence between two figures such that

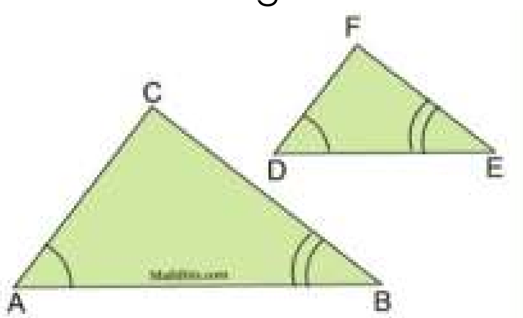
1) All corresponding **angles** are **congruent**
2) All corresponding **sides** are **proportional**

DILATIONS CREATE SIMILAR FIGURES!

PROVING SIMILAR FIGURES:

Key Idea #1: Two figures are similar if and only if there exists a dilation that will map one figure onto the other

Key Idea #2: There are 3 Triangle Similarity Theorems that may be used to prove two similar triangles:

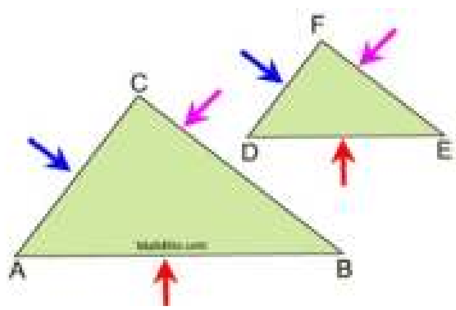


AA~

If: $\angle A \cong \angle D$ and $\angle B \cong \angle E$

Then: $\Delta ABC \sim \Delta DEF$

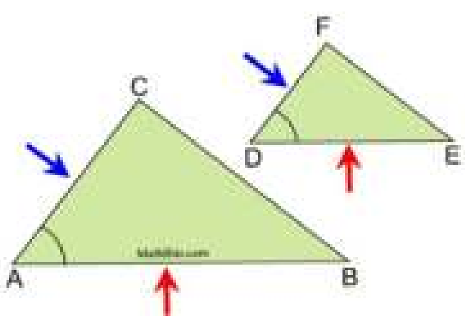
SSS~



If: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Then: $\Delta ABC \sim \Delta DEF$

SAS~



If: $\frac{AB}{DE} = \frac{AC}{DF}$

and $\angle A \cong \angle D$

Then: $\Delta ABC \sim \Delta DEF$

More Properties of Similar Figures

Ratio of Perimeters, Altitudes, Medians, Diagonals, & Angle Bisectors:

If two polygons are **similar**, their corresponding sides, altitudes, medians, diagonals, angle bisectors and perimeters are all in the same ratio.

$$\frac{side\ 1}{side\ 2} = \frac{perimeter\ 1}{perimeter\ 2}$$

Ratio of Areas:

If two polygons are **similar**, the ratio of their areas is equal to the square of the ratio of their corresponding sides.

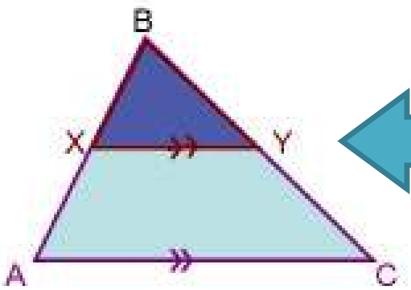
$$\frac{(side\ 1)^2}{(side\ 2)^2} = \frac{area\ 1}{area\ 2}$$

Ratio of Volumes:

If two polygons are **similar**, the ratio of their volumes is equal to the cube of the ratio of their corresponding sides.

$$\frac{(side\ 1)^3}{(side\ 2)^3} = \frac{volume\ 1}{volume\ 2}$$

Side Splitter Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



Anytime you see a smaller triangle in a larger triangle with a shared angle, look to see if the triangles are similar by AA~! If the triangles are similar, then the sides are proportional!

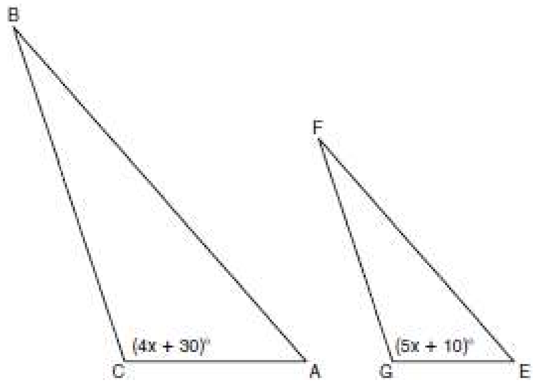
The small triangle is similar to the larger triangle by AA~.

The reflexive property and parallel lines creating congruent corresponding angles could be used to prove similarity.

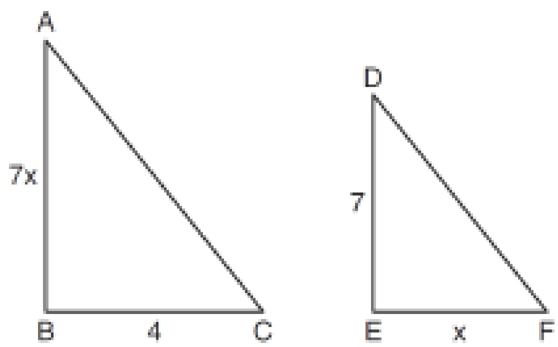
PRACTICE PROBLEMS

1. In the diagram below, $\Delta ABC \sim \Delta EFG$, $m\angle C = 4x + 30$, and $m\angle G = 5x + 10$. Determine the value of x.

Need help? Scan:



2. As shown in the diagram below, $\Delta ABC \sim \Delta DEF$, $AB = 7x$, $BC = 4$, $DE = 7$, and $EF = x$.



What is the length of $\overline{AB}$?

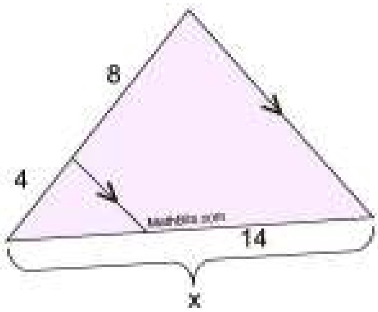
1) 28
2) 2
3) 14
4) 4

3. Given $\triangle ABC \sim \triangle DEF$ such that $\frac{AB}{DE} = \frac{3}{2}$. Which statement is *not* true?

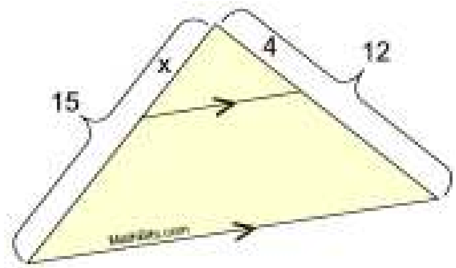
1) $\frac{BC}{EF} = \frac{3}{2}$
2) $\frac{m\angle A}{m\angle D} = \frac{3}{2}$
3) $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEF} = \frac{9}{4}$
4) $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{3}{2}$

4. A triangle has sides with lengths 6, 8 and 11 inches. A second triangle has sides with lengths 18, 16, and 22 inches. Are these triangles similar?

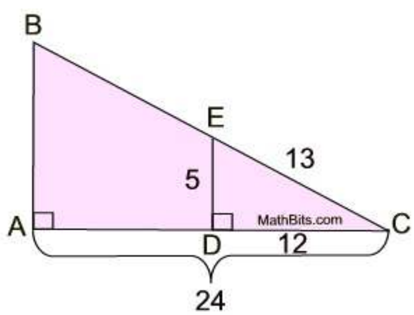
5. Find x.



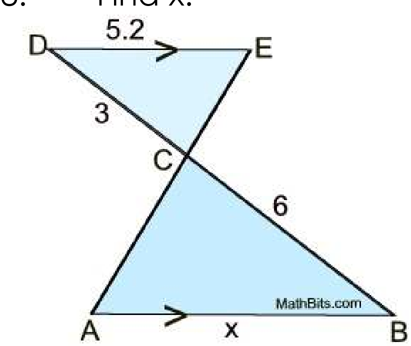
6. Find x.



7. Find AB.



8. Find x.



9.If AB = 4, BC = 12, DE = 3, and EF = 9, and $< B \cong < E$, are triangles ABC and DEF similar?

Need help?
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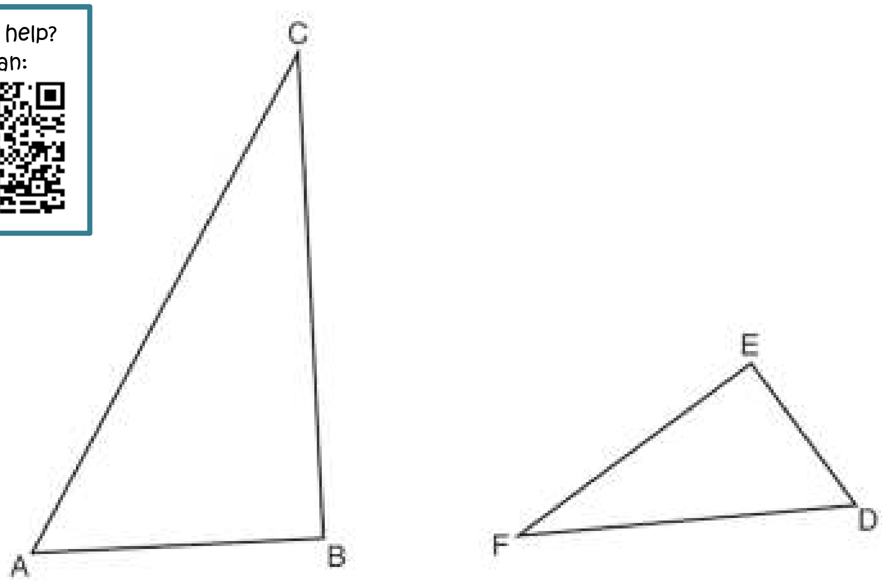


10. If ΔABC is dilated by a scale factor of 5, which statement is true of the image $\Delta A'B'C'$?

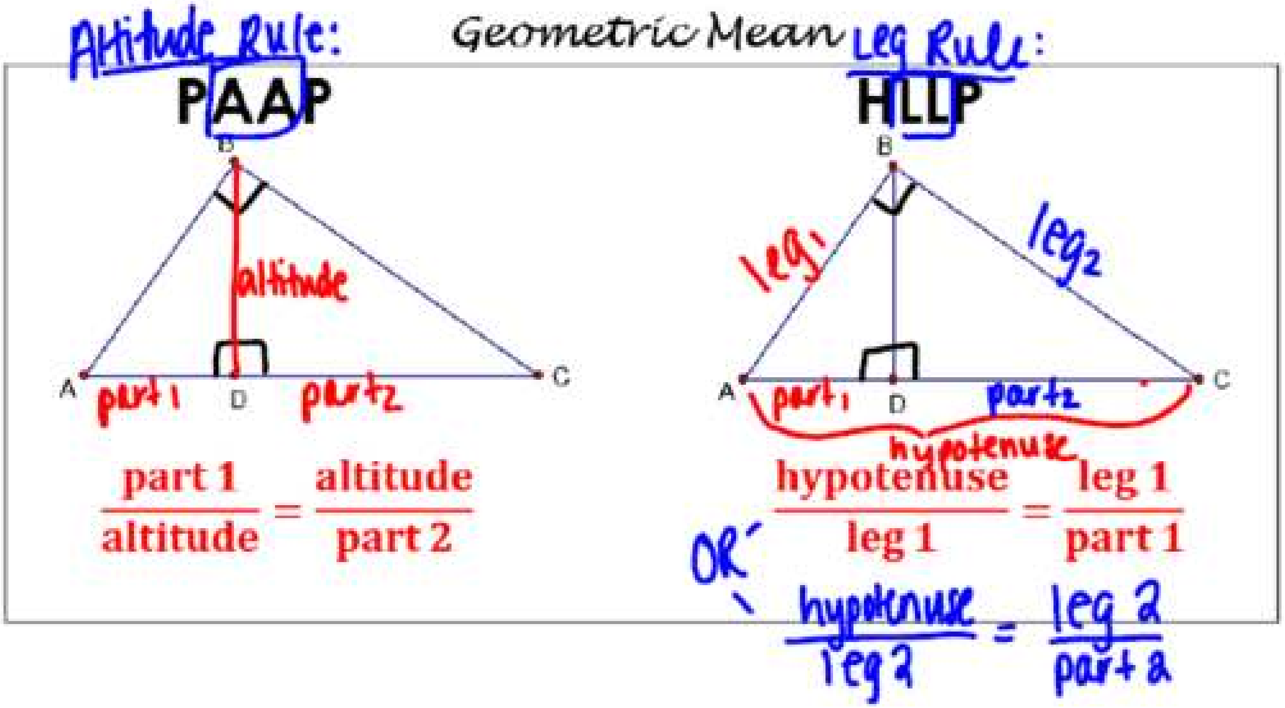
(1) $5A'B' = AB$

(2) $B'C' = 5BC$

(3) $m < A' = 5(m < A)$

(4) $5(m < C') = m < C$

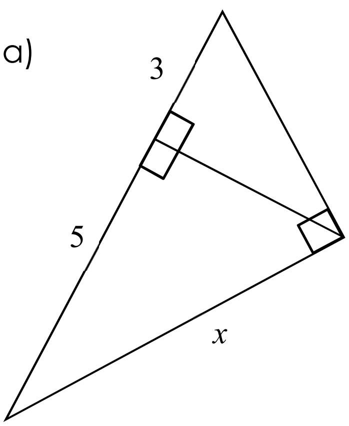
11. A tree casts a shadow 20 feet long. Jake stands at a distance of 12 feet from the base of the tree, such that the end of Jake's shadow meets the end of the tree's shadow. If Jake is 6 feet tall, determine and state the height of the tree to the nearest tenth of a foot.



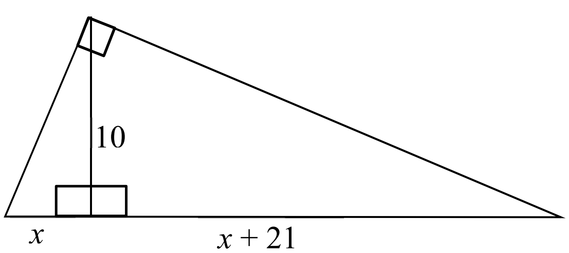
EXAMPLE:
Find x in each of the triangles below.

Need help? Scan:

a)



b)



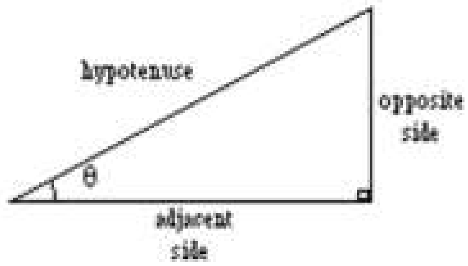
Unit 6: Right Triangle Trigonometry

Finding the Missing Side and/or Angle of a Right Triangle

SOH $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

CAH $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

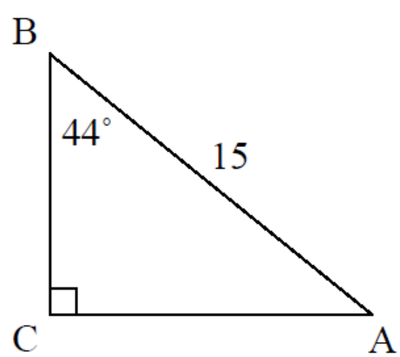
TOA $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$



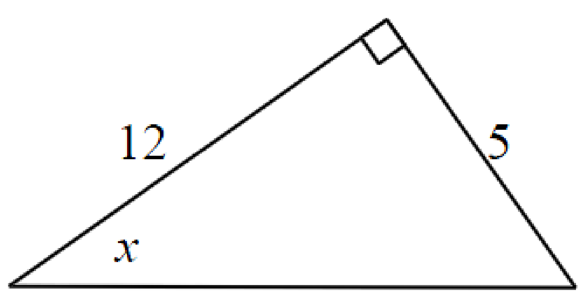
Need help? Scan:

Examples:

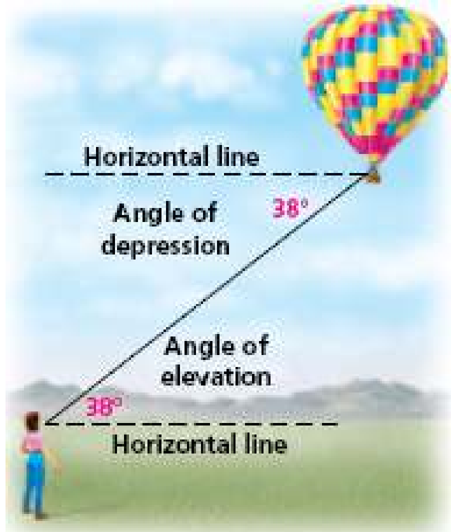
1. In the right triangle below, find the length of $\overline{AC}$ to the nearest tenth.



2. In the right triangle below, find the measure of x to the nearest tenth of a degree.



Trig Applications



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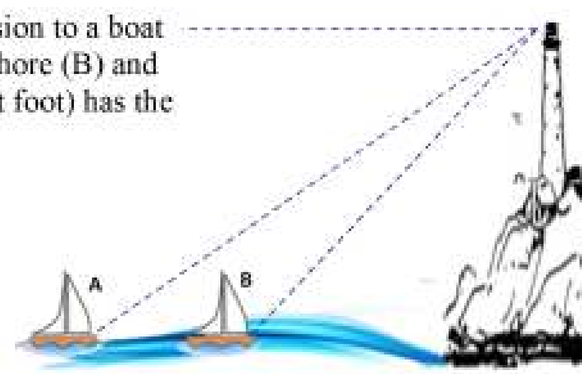
EXAMPLE 1:

A man who is 5 feet 9 inches tall casts a shadow of 8 feet 6 inches. Assuming that the man is standing perpendicular to the ground, what is the angle of elevation from the end of the shadow to the top of the man's head, to the *nearest tenth of a degree*?

EXAMPLE 2:

From a lighthouse 460 ft above sea level, the angle of depression to a boat (A) is 42º. Sometime later the boat has moved closer to the shore (B) and the angle of depression measures 50º. How far (to the nearest foot) has the boat moved in that time?



EXAMPLE 3:

Two people are 27 ft apart. Jeff who is farthest away from the building sees the top of the building at 33° and Jessica sees the top of the building at 42°. What is the height of building (to the nearest foot)?

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Need help? Scan:

SINE & COSINE COMPLEMENTS

If $m < A + m < B = 90°$, then $\sin A = \cos B$.

EXAMPLE 1: Explain why $\cos(x) = \sin(90 - x)$ for x such that $0 < x < 90$.

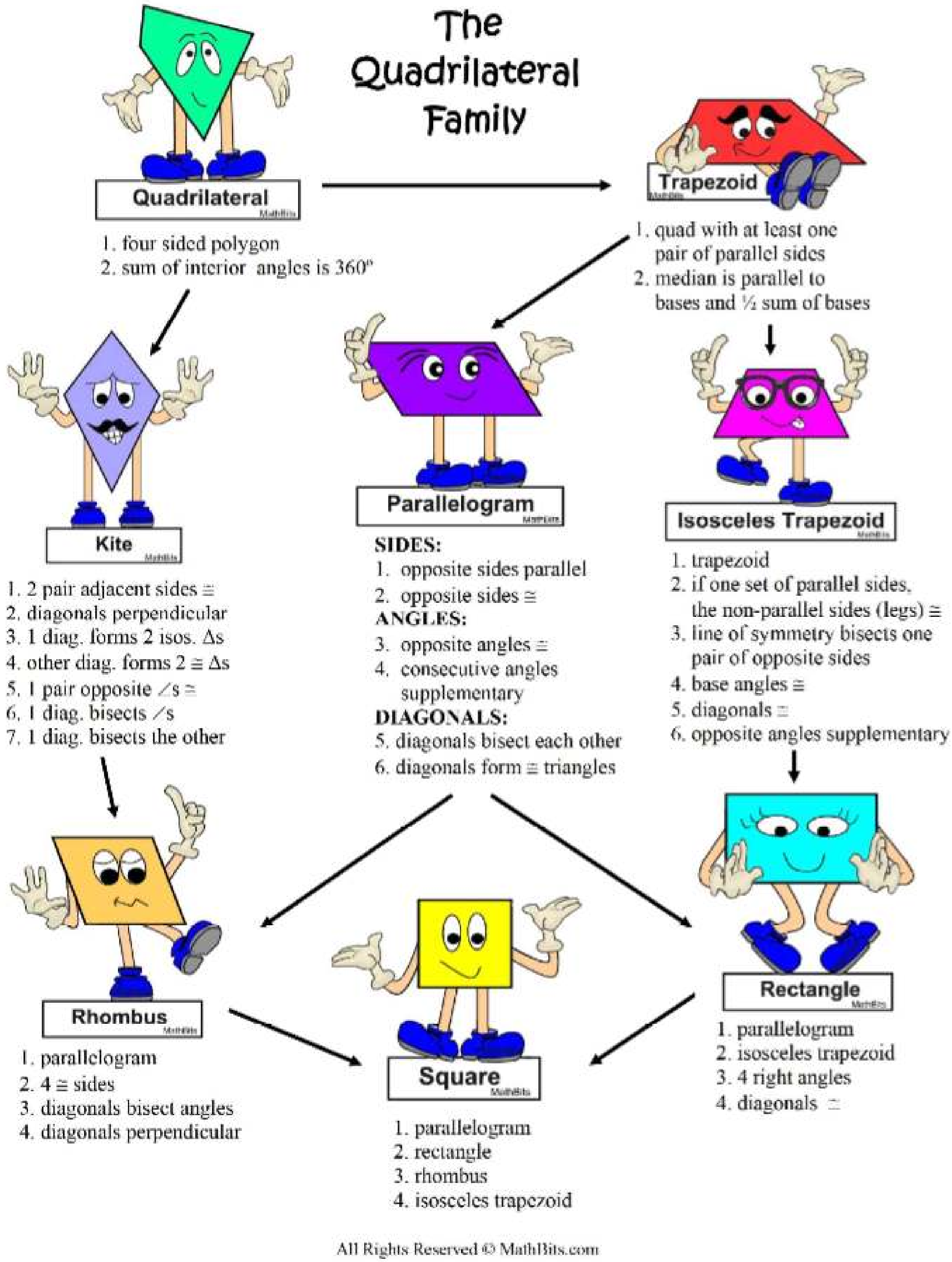
EXAMPLE 2:

In right triangle ABC with the right angle at C, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of x. Explain your answer.

EXAMPLE 3: Solve for θ:

$$\sin\left(\frac{\theta}{3} + 10\right) = \cos\theta$$

Unit 7: Quadrilaterals



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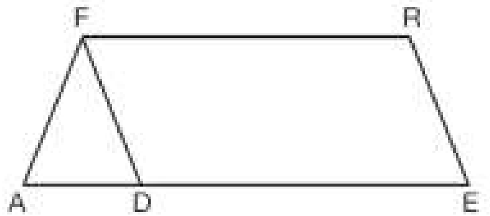
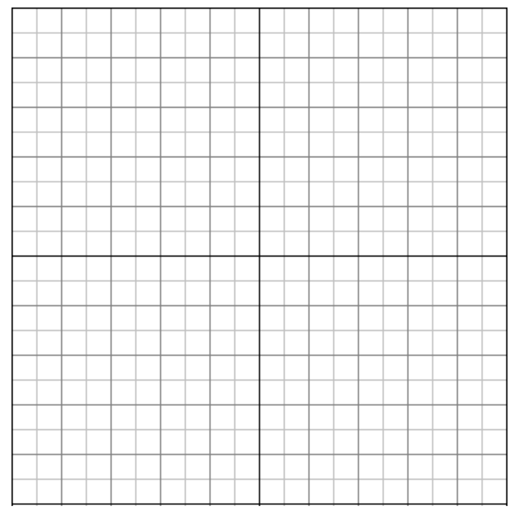
Examples:

1. In the diagram of parallelogram $FRED$ shown below, $\overline{ED}$ is extended to A, and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.

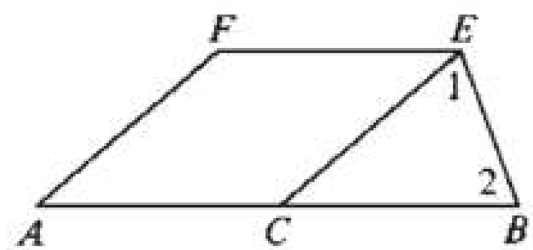


If $m < R = 122°$, what is $m < AFD$?

2. Given quadrilateral PQRS with P(0, 2) Q(4, 8) R(7, 6) S(3, 0)
Prove quadrilateral PQRS is a rectangle



3. Given: ACEF is a parallelogram
$\overline{AC} \cong \overline{BC}$ and $\angle 1 \cong \angle 2$
Prove: ACEF is a rhombus



Unit 8: Two-Dimensional Shapes

Constructing Inscribed Polygons

Regular Hexagon Inscribed in Circle O

Need help?
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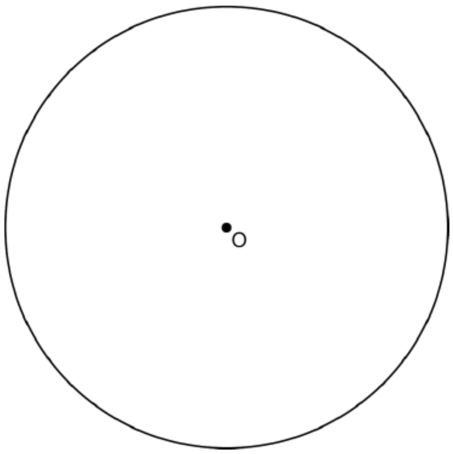
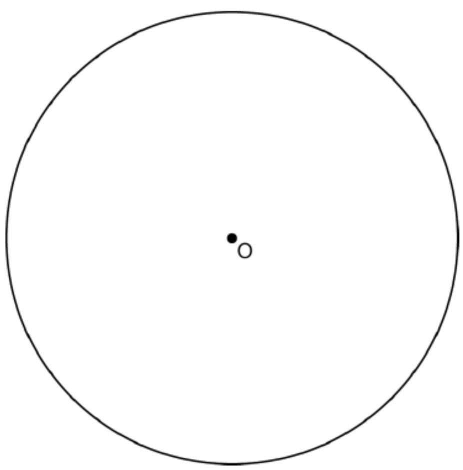


Steps:
1. Place point P anywhere on the circle's circumference.
2. Measure the length of the radius, OP.
3. WITHOUT CHANGING THE COMPASS, start at point P and draw another arc on the circle. Then, place the compass on that point and draw another arc on the circle. Repeat this process until you get back to point P.
4. Connect all 6 points.

Equilateral Triangle Inscribed in Circle O



Steps:
1. Start with the same process used for constructing a regular hexagon inscribed in a circle.
2. Once you get all 6 points, connect every other point instead of connecting all 6.

Square Inscribed in Circle O

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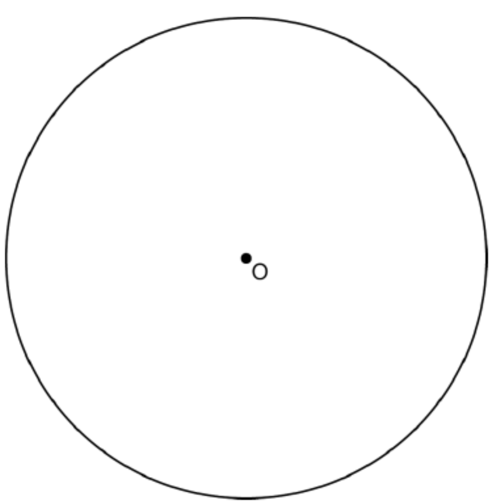

Steps:
1. Draw a diameter.
2. Construct the perpendicular bisector of the diameter.
3. Label the points where the bisector intersects the circle.
4. Connect all 4 points.

Perimeter on the Coordinate Plane

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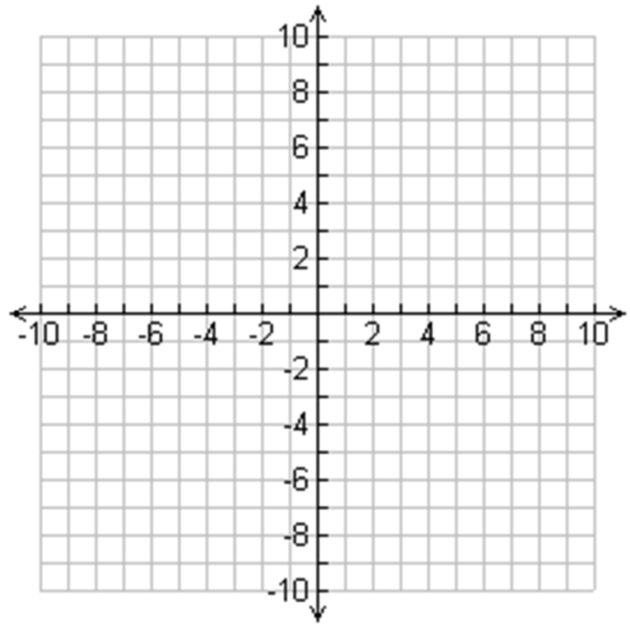
Example:

Given ∆ABC, A (-3,4) B (1,7) C (7,-1), determine the perimeter.



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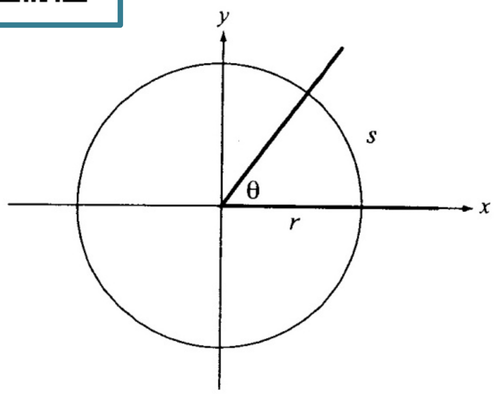
Arc Length



θ = central angle in <u>radians</u>

r = radius
(in, cm, ft, m, etc)

s = arc length
(in, cm, ft, m, etc)

$$s = \theta \cdot r$$

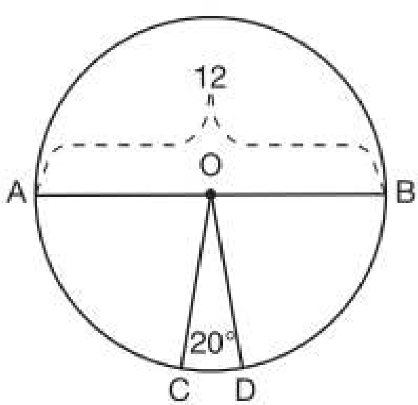
EXAMPLE: Find to the nearest tenth the length of the arc of a circle with a radius of 6 yards and intercepted by a central angles measuring 270 degrees.

Area of a Sector

$$\textit{sector area} = \left(\frac{\textit{central angle}}{360°}\right) \cdot \pi r^2$$

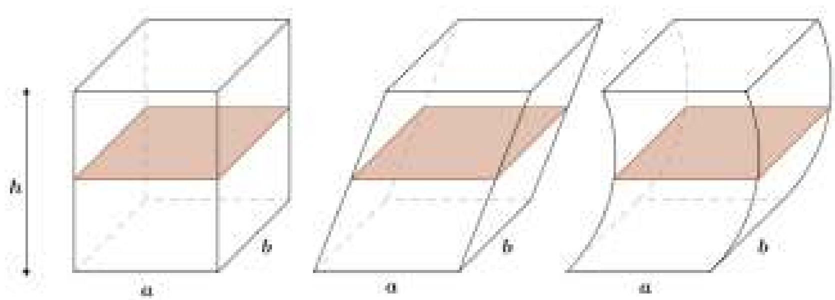
EXAMPLE: In the diagram below of circle O, diameter $\overline{AB}$ and radii $\overline{OC}$ and $\overline{OD}$ are drawn. The length of $\overline{AB}$ is 12 and the measure of $\angle COD$ is 20 degrees.

If $\widehat{AC} \cong \widehat{BD}$, find the area of sector BOD in terms of π.



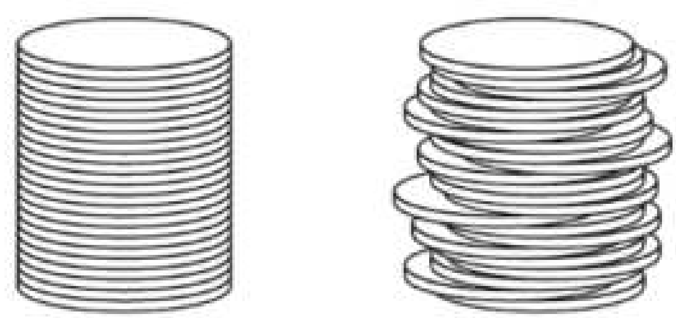
CAVALIERI'S PRINCIPLE:

Given two solids that are included between two parallel planes, if every plane parallel to the two planes intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.



Example:

Two stacks of 23 quarters each are shown below. One stack forms a cylinder but the other stack does not form a cylinder.

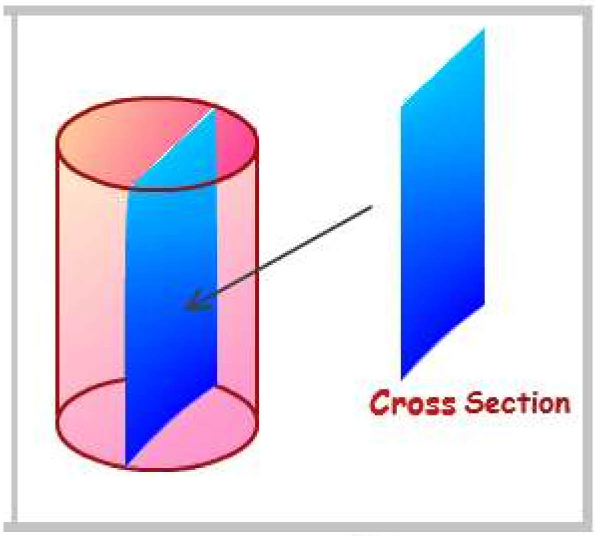


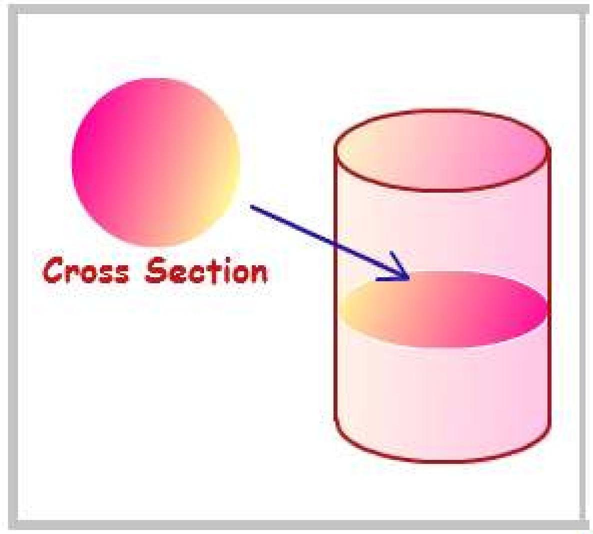
Use Cavelieri's principle to explain why the volumes of these two stacks of quarters are equal.

The two stacks only contain quarters, so the area of each cross section is the same. Since the two stacks contain 23 quarters, the heights of the stacks are equal. Thus, by Cavelieri's principle, the volumes of the stacks are equal.

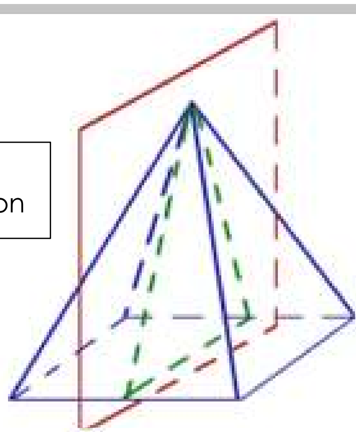
Cross Sections

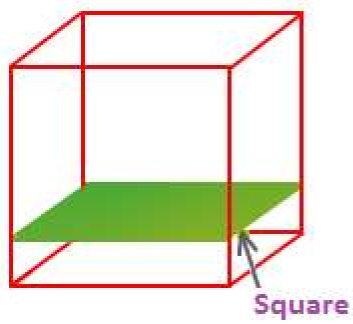
Cross Section: the intersection of a 3D figure with a plane; a "slice"

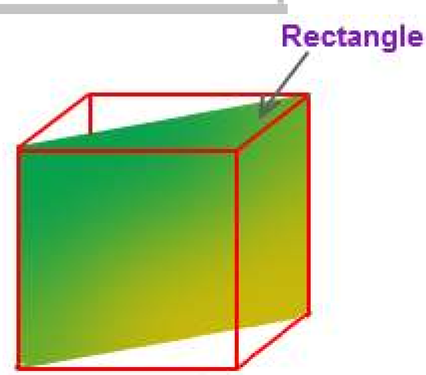




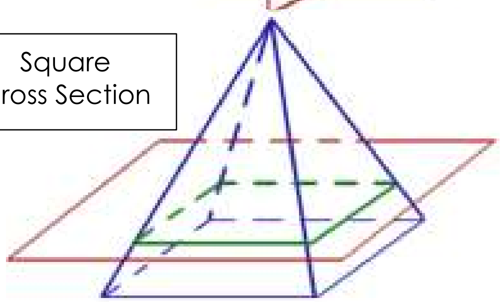
Triangle Cross Section

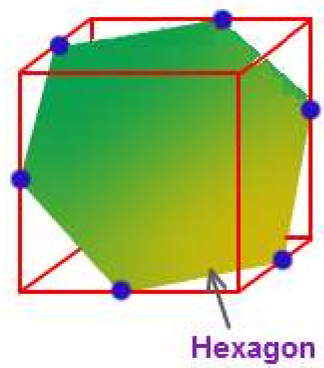




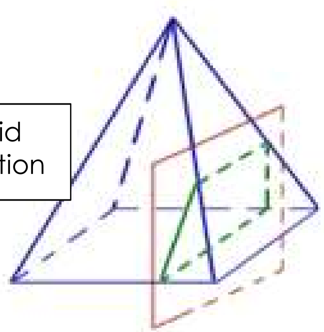


Square Cross Section





Trapezoid Cross Section



Cross Section Property:

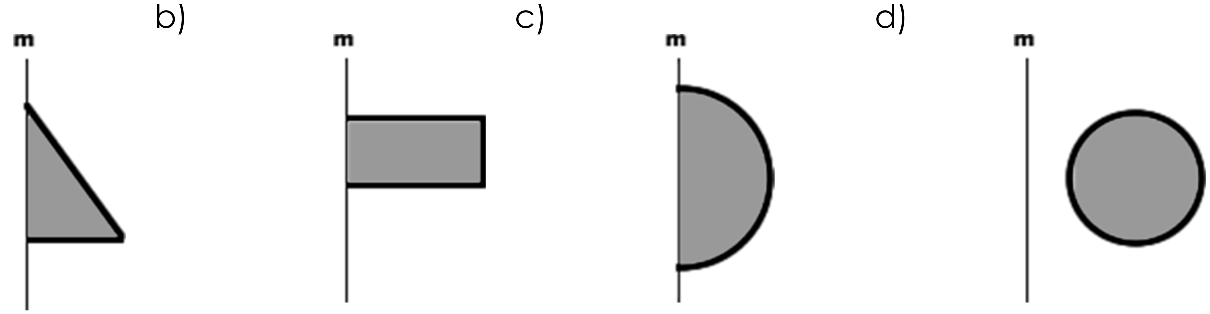
# of sides of cross section ≤ # of faces of 3D solid

Rotations of 2D Shapes

Example 1:

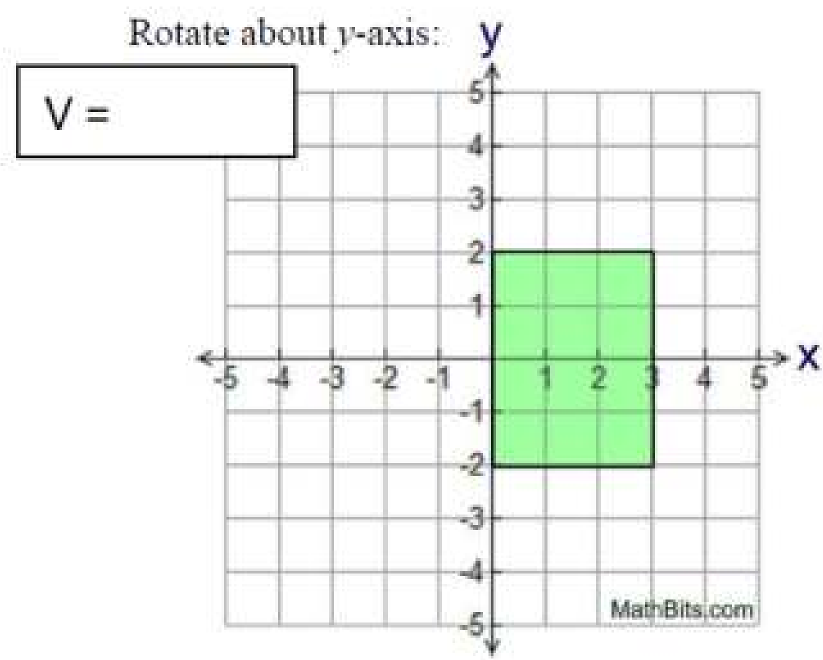
Describe the solid that is formed by rotating each of these figures about line m and sketch it.

a) b) c) d)



Name/Description Name/Description Name/Description Name/Description

Example 2:



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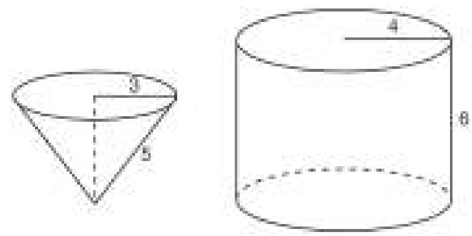
3D Applications

Example:

In the diagram below, a right circular cone with a radius of 3 inches has a slant height of 5 inches, and a right cylinder with a radius of 4 inches has a height of 6 inches.



Determine and state the number of full cones of water needed to completely fill the cylinder with water.

Density Problems

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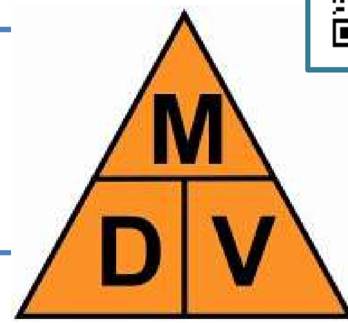
<u>Density:</u> mass per unit volume

$$Density = \frac{Mass}{Volume} \quad \text{or} \quad D = \frac{M}{V}$$



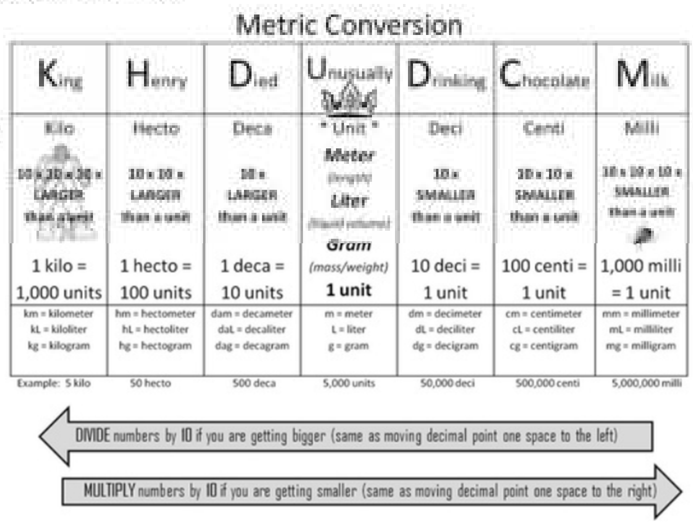
Example 1:

A shipping container is in the shape of a right rectangular prism with a length of 12 feet, a width of 8.5 feet, and a height of 4 feet. The container is completely filled with contents that weigh, on average, 0.25 pound per cubic foot. What is the weight, in pounds, of the contents in the container?

Example 2:

A contractor needs to purchase 500 bricks. The dimensions of each brick are 5.1 cm by 10.2 cm by 20.3 cm, and the density of each brick is 1920 kg/m^3.

The maximum capacity of the contractor's trailer is 900 kg. Can the trailer old the weight of 500 bricks? Justify your answer.



Metric Conversion

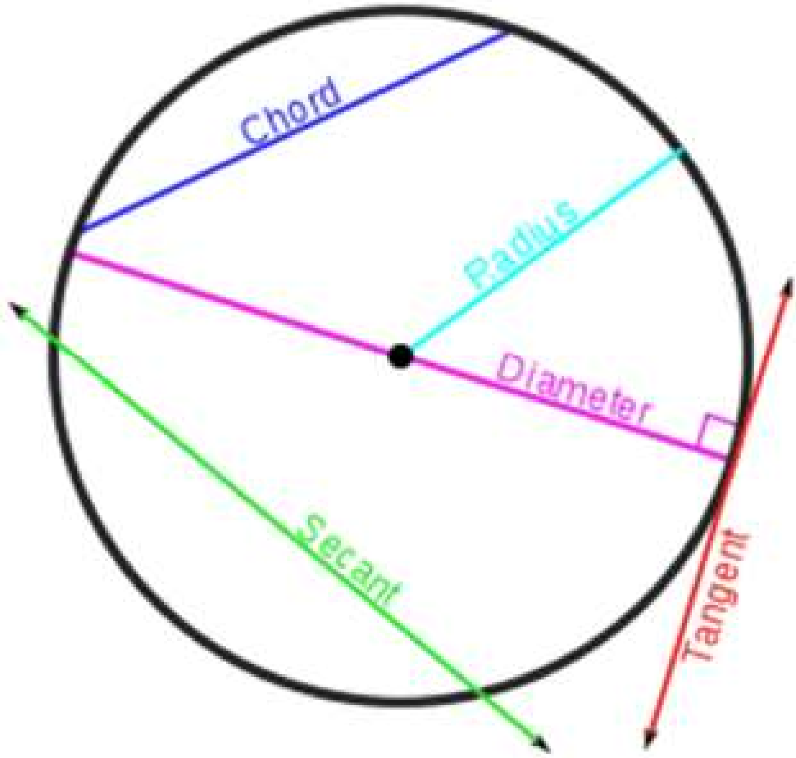
King	Henry	Died	Unusually	Drinking	Chocolate	Milk
Kilo	Hecto	Deca	* Unit *	Deci	Centi	Milli
10 x 10 x 10 x LARGER than a unit	10 x 10 x LARGER than a unit	10 x LARGER than a unit	Meter (length) Liter (liquid volume) Gram (mass/weight)	10 x SMALLER than a unit	10 x 10 x SMALLER than a unit	10 x 10 x 10 x SMALLER than a unit
1 kilo = 1,000 units	1 hecto = 100 units	1 deca = 10 units	1 unit	10 deci = 1 unit	100 centi = 1 unit	1,000 milli = 1 unit
km = kilometer kL = kiloliter kg = kilogram	hm = hectometer hL = hectoliter hg = hectogram	dam = decameter daL = decaliter dag = decagram	m = meter L = liter g = gram	dm = decimeter dL = deciliter dg = decigram	cm = centimeter cL = centiliter cg = centigram	mm = millimeter mL = milliliter mg = milligram
Example: 5 kilo	50 hecto	500 deca	5,000 units	50,000 deci	500,000 centi	5,000,000 milli

DIVIDE numbers by 10 if you are getting bigger (same as moving decimal point one space to the left)

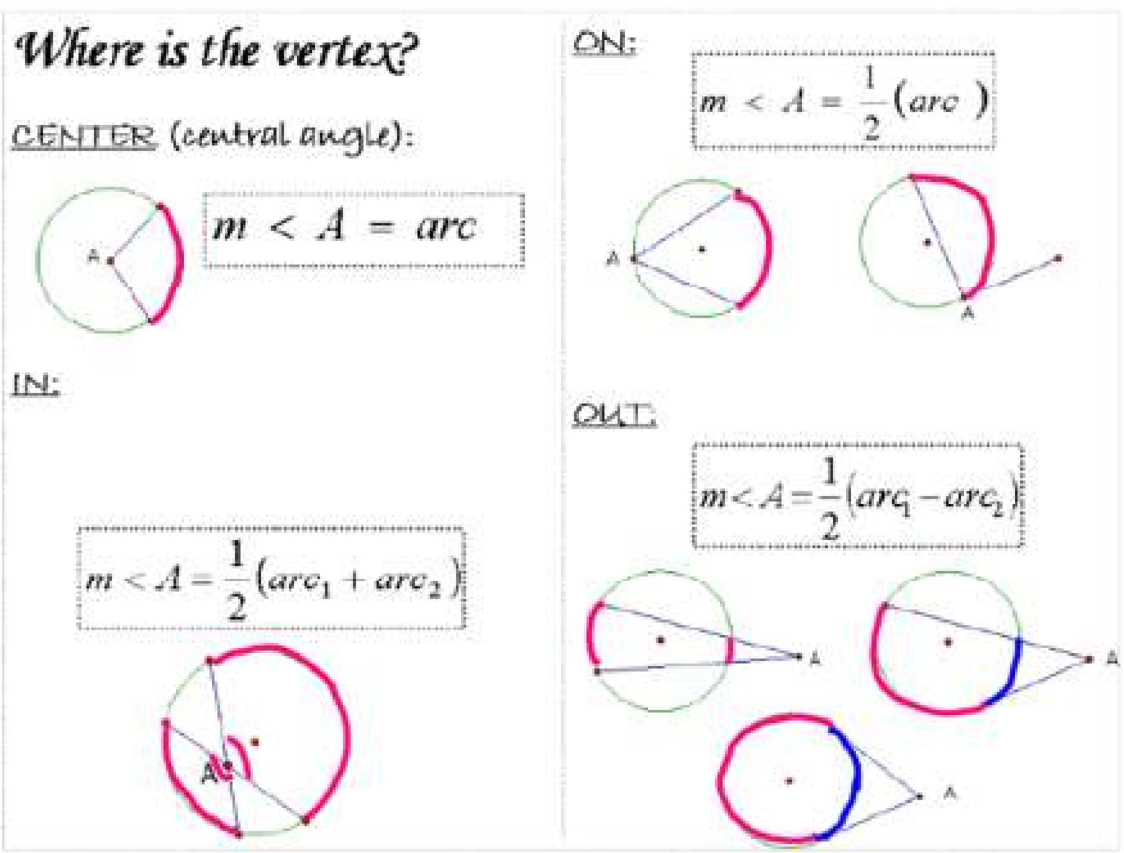
MULTIPLY numbers by 10 if you are getting smaller (same as moving decimal point one space to the right)

Unit 10: Circles

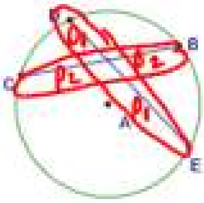
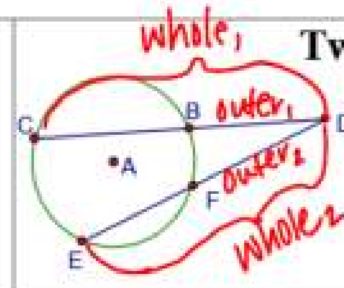
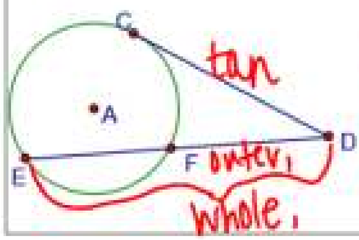
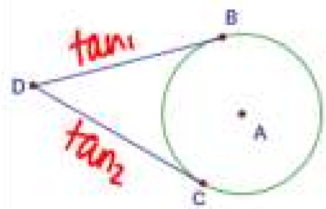
I. Vocab



II. Angles



III. Segments

Two Chords	Two Secants
 $p_1 \cdot p_1 = p_2 \cdot p_2$	 $o_1 \cdot W_1 = o_2 \cdot W_2$
Secant & Tangent	**Two Tangents**
 $o_1 \cdot W_1 = tan^2$	 $tan_1 = tan_2$

IV. Circle Proofs

DON'T FORGET: In a circle, all radii are congruent.

Theorems about Circles

1. If two inscribed angles of a circle intercept the same arc, then they are congruent.

∠F ≅ ∠D
∠C ≅ ∠E

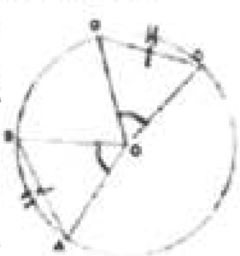
2. An angle inscribed in a semi-circle is a right angle.

m∠K = 90°

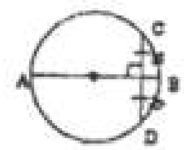
3. In a circle, congruent central angles cut congruent arcs & have congruent chords.

4. In a circle, congruent arcs have congruent central angles & congruent chords.

5. In a circle, congruent chords have congruent central angles & congruent arcs.



6. If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc



7. Two chords are congruent if and only if they are equidistant from the center.

$\overline{AB} \cong \overline{CD}$

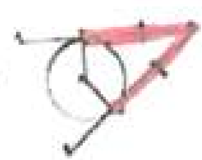
8. If two chords are parallel, then they intercept congruent arcs.

$\overset{\frown}{AC} \cong \overset{\frown}{BD}$

9. A line is tangent to a circle if and only if it is perpendicular to a radius at its point of intersection with the circle.

$\overline{OR} \perp \overline{ST}$

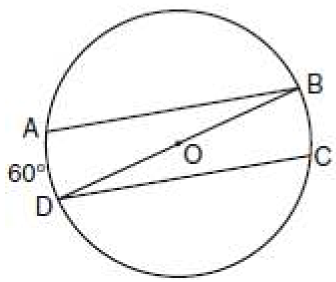
10. Two tangent segments to a circle from the same exterior point have equal lengths.



$\overline{BC} \cong \overline{DC}$

Practice Problems

1. In the diagram of circle O below, chords $\overline{AB}$ and $\overline{CD}$ are parallel, and $\overline{BD}$ is a diameter of the circle. If $m\overset{\frown}{AD} = 60$, what is $m\angle CDB$?



1) 20
2) 30
3) 60
4) 120

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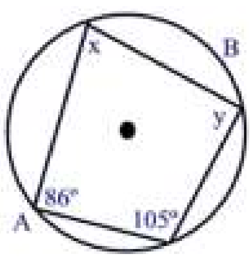
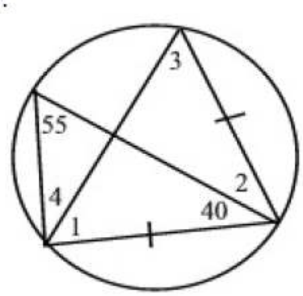
2. Given circle with center indicated and inscribed quadrilateral. Find x and y.

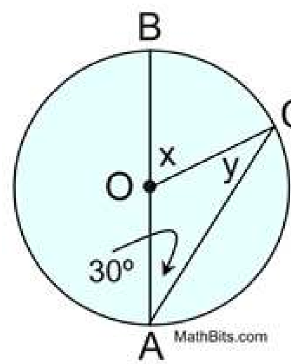


3.

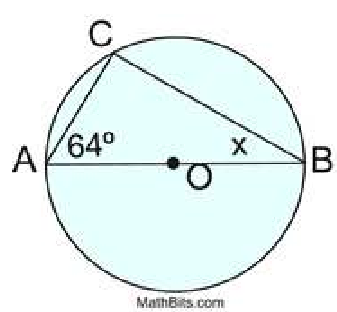


m<1 = ________
m<2 = ________
m<3 = ________
m<4 = ________

4. Given circle O with diameter $\overline{AB}$. Find x and y.



5. Given circle O with diameter $\overline{AB}$. Find x.



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Graphs & Equations of Circles

Equation of a Circle

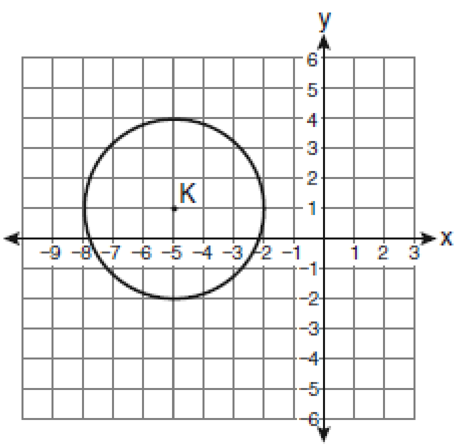
$$(x-h)^2+(y-k)^2=r^2$$

Center: (h,k)

Radius: r

1. What is an equation of a circle with its center at $(-3,5)$ and a radius of 4?
   1) $(x-3)^2+(y+5)^2=16$
   2) $(x+3)^2+(y-5)^2=16$
   3) $(x-3)^2+(y+5)^2=4$
   4) $(x+3)^2+(y-5)^2=4$

2. Which equation represents circle *K* shown in the graph below?



   1) $(x+5)^2+(y-1)^2=3$
   2) $(x+5)^2+(y-1)^2=9$
   3) $(x-5)^2+(y+1)^2=3$
   4) $(x-5)^2+(y+1)^2=9$

3. What are the center and the radius of the circle whose equation is $(x-3)^2+(y+3)^2=36$
   1) center $=(3,-3)$; radius $=6$
   2) center $=(-3,3)$; radius $=6$
   3) center $=(3,-3)$; radius $=36$
   4) center $=(-3,3)$; radius $=36$

Completing the Square to find the Center & Radius

Convert $x^2 + y^2 - 4x - 6y + 8 = 0$ into center-radius form.

When given the "general form", it will be necessary to covert the equation into the *center-radius form* to determine the center and the radius and to graph the circle. To accomplish this conversion, you will need to "complete the square" on the equation.

We will be creating two perfect square trinomials within the equation.

$$x^2 + y^2 - 4x - 6y + 8 = 0$$

$$x^2 - 4x + y^2 - 6y = -8$$

$$x^2 - 4x + \square + y^2 - 6y + \square = -8 + \square + \square$$

$$x^2 - 4x + \boxed{4} + y^2 - 6y + \boxed{9} = -8 + \boxed{4} + \boxed{9}$$

$$(x-2)^2 + (y-3)^2 = 5$$

The **center** of this circle is at (2,3)

and the **radius** is $\sqrt{5}$.

- Start by grouping the *x*-related terms together and the *y*-related terms together. Move any numerical constants (plain numbers) to the other side.
- Get ready to insert the needed values for creating perfect square trinomials. Remember to balance both sides of the equation.
- Find the missing value by taking half of the "middle term" (the linear coefficient) of the trinomial and squaring it. This value will always be positive as a result of the squaring process.
- Rewrite in factored form.

EXAMPLE: Find the coordinates of the center of the circle and its radius.

$$x^2 + y^2 + 2x - 4y - 11 = 0$$

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